

# The nature of information and its effect on bidding behavior: laboratory evidence in a first price common value auction \*

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## **Abstract**

*We study in the laboratory a series of first price sealed bid auctions of a common value good. Bidders face three types of information: private information, public information and common uncertainty. Auctions are characterized by the relative size of these three information elements. Half of our subjects bid differently depending on whether the last piece of information obtained is private or public but they do not react to each type of information as predicted by theory. The other half of the subjects do not distinguish between private and public information and either consistently underbid or consistently overbid.*

Keywords: laboratory experiments, common value auctions, winner's curse.

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# 1 Introduction

Common value auctions have been extensively studied in the laboratory. Two major findings are behavioral heterogeneity (Crawford and Iriberry, 2007) and the pervasiveness of the winner's curse (Kagel and Levin (1986, 2008)). Despite the existing literature, our knowledge of bidding behavior in those games is still incomplete. The goal of this paper is to improve such understanding. To this purpose we introduce two novel features in the design of an otherwise standard first price sealed bid common value auction with two bidders. First, we assume that the value of the good is the sum of  $N$  independent components and that each bidder observes the content of a subset of these components. Subjects always know which components are observed by the other bidder. Therefore, there are three clearly identified possible types of information in the game: private information (the components observed by only one bidder), public information (the components observed by both bidders) and common uncertainty (the components observed by no bidder). Second, we vary the number of components observed by each bidder, which affects the information structure in the auction. We consider five different structures: two with private information and common uncertainty, two with private information and public information, and one with private information only. Assuming risk neutrality, we show that the Nash Equilibrium (NE) bid in this auction is the sum of different elements each reflecting one type of information. With respect to private information, bidders shade their bids like in a typical common value auction (see e.g. Milgrom and Weber (1982)). With respect to common uncertainty and public information, bidders compete à la Bertrand and bid the expected value and the realized value respectively.

This auction is run in the laboratory and the experimental data is analyzed in different ways. First, we perform descriptive statistics of aggregate bids, aggregate payoffs, and changes in aggregate bids as we vary the information structure. We show that, on average, bidders do not distinguish correctly between the three different types of information. In our experiment, half of the subjects change their bids in a similar way whether the new piece of evidence is observed privately or observed also by the other bidder. This suggests that subjects do not make a rational strategic use of private information, resulting in deviations from NE predictions. Our aggregate analysis also reveals a large dispersion, implying a high level of behavioral heterogeneity in our population. To study this issue in more detail, we then conduct a cluster analysis. Since deviations from NE predictions seem to be driven by a wrong understanding of the information structure, we compute for each subject the average deviation from the NE prediction in each information structure. Given this new

dataset, we conduct a model-based clustering method to endogenously determine clusters of individuals. This method reveals the existence of 6 distinct clusters in our population, differing in the size of their departures from NE as a function of the information structure. The analysis of each cluster separately reveals that heterogeneity across individuals is largely due to their different comprehension of the information structure. Only 63% of our subjects (clusters 1, 2 and 5) bid relatively close to equilibrium. Of these, 46% (clusters 1 and 2) realize the existence of the different types of information. They bid differently depending on whether the new information is private or public, although they still exhibit deviations from Nash: they overbid common uncertainty and underbid public information. The other 17% (cluster 5) have a much more imperfect grasp of the different types of information. Finally, 37% of the subjects (clusters 3, 4 and 6) hardly differentiate between public and private information. They also consistently overbid or consistently underbid.

Our analysis relates to two strands of the experimental literature: common value auctions and auctions with variable amounts of information. Kagel and Levin (1986) is the classical reference on common value auctions in the laboratory. They assume the value of the good is drawn from some distribution (typically uniform). Bidders receive a signal which is drawn from another distribution centered around the true realization. We use a slightly different model where the value of the good is the sum of several independent signals, and each signal may or may not be observed by bidders. This is formally closer to Albers and Harstad (1991), Avery and Kagel (1997), and Klemperer (1998).<sup>1</sup> As noted above, the novelty of our paper lies in explicitly modeling different types of information and varying their relative importance.

The experimental literature that varies the amount of information in auction settings is also related. Andreoni et al. (2007) study a series of private value auctions in which bidders know not only their own valuation but also the valuation of some other bidders. Naturally, the private value setting precludes any winner's curse problem. Mares and Shor (2008) analyze common value auctions with constant informational content but distributed among a varying number of bidders. The paper explores the trade-off competition vs. precision of estimates. Grosskopf et al. (2010) experimentally investigate the role of asymmetric

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<sup>1</sup>In the first study the good is the sum of the signals and each bidder observes only one signal, hence the number of bidders is equal to the number of signals. In the last two studies, each bidder has one private signal. The value of the good is the sum of the signals for one bidder and the sum of the signals plus a private value component for the other bidder. Therefore, when the private value component is zero, their model is equivalent to our treatment with only private information.

information by varying the number of bidders who receive a signal about the common value of the good. Like our study, they find that the winner’s curse increases with private information. However, they do not study how the existence of other types of information may affect the bidding strategy of subjects. Finally, in our recent companion paper (Brocas et al. (2014)) we study a similar problem than here using a second price auction and a slightly different design. The objective is to determine whether the imperfect differential treatment of private and public information also occurs under alternative mechanisms. The answer is affirmative: we also find that in second-price auctions subjects differentiate insufficiently between private and public information. The paper however focuses on the study of individual strategies to explain the deviations from Nash equilibrium rather than a cluster analysis to understand common patterns of choice.

The paper proceeds as follows. The theoretical framework is briefly described in section 2. The experimental setting is developed in section 3. The aggregate analysis of the experimental data (aggregate bids, aggregate payoffs and changes in aggregate bids as a function of the type of information revealed) is discussed in section 4. The cluster analysis and the regression analysis are performed in section 5. Final conclusions are presented in section 6. Proofs, tables, figures and a sample copy of instructions are gathered in the Appendix.

## 2 Theoretical model

Consider a single good made of  $N$  components (with  $N$  even and greater than or equal to four). Each component  $i \in \{1, \dots, N\}$  has a value  $x_i$  independently drawn from a continuous distribution with positive density  $g(x_i)$  on  $[\underline{x}, \bar{x}]$  and cumulative distribution  $G(x_i)$ . The total value of the good is the same for every individual and equal to the sum of the components,  $V = \sum_{i=1}^N x_i$ .

Two risk neutral bidders,  $A$  and  $B$  indexed by  $j$ , bid for this good in a first price sealed bid auction with no reserve price. Before placing their bids,  $A$  observes the first  $r$  components of the good,  $\{x_1, \dots, x_r\}$ , and  $B$  observes the last  $r$  components of the good,  $\{x_{N-r+1}, \dots, x_N\}$ , where  $r \in \{1, \dots, N-1\}$ .

Note that each bidder observes exactly  $r$  components and does not observe exactly  $N-r$  components. Each bidder knows which components are and are not observed by the other bidder. Under this formalization, bidders always have *private information*: by construction, some components are only observed by  $A$  (e.g.,  $x_1$ ) while other components

are only observed by  $B$  (e.g.,  $x_N$ ). It is useful to split the auction into three cases. When  $r < N/2$ , there is *common uncertainty* (on top of private information): the components  $\{x_{r+1}, \dots, x_{N-r}\}$  are not observed by any bidder. When  $r > N/2$ , there is *public information* (on top of private information): the components  $\{x_{N-r+1}, \dots, x_r\}$  are observed by both bidders. Finally, when  $r = N/2$ , there is neither common uncertainty nor public information, and the value of the good is the sum of the private information of both bidders. For the rest of the analysis we introduce the following notations.

- $X_A^r = \sum_{i=1}^{\min\{r, N-r\}} x_i$ : the sum of A's private information.
- $X_B^r = \sum_{i=\max\{N-r+1, r+1\}}^N x_i$ : the sum of B's private information.
- $E[X_\emptyset^r] = \sum_{i=r+1}^{N-r} E[x_i]$ : the expected common uncertainty when  $r < N/2$ .
- $X_{\text{Pub}}^r = \sum_{i=N-r+1}^r x_i$ : the sum of public information when  $r > N/2$ .

When  $r \leq N/2$ , the private information of each bidder is an independent random variable with cumulative distribution  $F^r(\cdot)$  and density  $f^r(\cdot)$ . When  $r \geq N/2$ , the private information of each bidder is an independent random variable with cumulative distribution  $F^{N-r}(\cdot)$  and density  $f^{N-r}(\cdot)$ . Given that each component  $x_i$  has distribution  $G(x_i)$  and components are independent, we have

$$F^r(X_A^r) = \int_{\underline{x}}^{\bar{x}} \dots \int_{\underline{x}}^{\bar{x}} G(X_A^r - x_1 - \dots - x_{r-1})g(x_1) \dots g(x_{r-1})dx_1 \dots dx_{r-1}.$$

and analogously for  $F^r(X_B^r)$ . Lemma 1 characterizes the optimal bidding strategies and equilibrium utilities in this auction as a function of  $r$ .

**Lemma 1. (Nash equilibrium)** *The unique symmetric equilibrium bidding function of agent  $j$  is:*

- $b^r(X_j^r) = E[X_\emptyset] + 2 \left( X_j^r - \frac{\int_{\underline{X}^r}^{X_j^r} F^r(S)dS}{F^r(X_j^r)} \right)$  when  $r < N/2$ ,
- $b^r(X_j^r) = 2 \left( X_j^r - \frac{\int_{\underline{X}^r}^{X_j^r} F^r(S)dS}{F^r(X_j^r)} \right)$  when  $r = N/2$ ,
- $b^r(X_j^r) = X_{\text{Pub}}^r + 2 \left( X_j^r - \frac{\int_{\underline{X}^{N-r}}^{X_j^r} F^{N-r}(S)dS}{F^{N-r}(X_j^r)} \right)$  when  $r > N/2$ .

and the equilibrium expected utility of agent  $j$  is:

- $U_j^r(X_j^r) = \int_{\underline{X}^r}^{X_j^r} F^r(s) ds$  when  $r \leq N/2$ ,
- $U_j^r(X_j^r) = \int_{\underline{X}^{N-r}}^{X_j^r} F^{N-r}(s) ds$  when  $r \geq N/2$ .

The model is an extension of the wallet game to the case with multiple signals. Albers and Harstad (1991) analyze the wallet game in a second price sealed bid auction with multiple bidders, each having exactly one signal, whereas Klemperer (1998) discusses the effect of small asymmetries in a two-bidders English auction wallet game. In our model, the optimal bidding function is the sum of two elements. The first element reflects common uncertainty when  $r < N/2$  and public information when  $r > N/2$ , while the second element reflects private information for all  $r$ . With regard to the first element, agents compete à la Bertrand and end up bidding the expected value of the common uncertainty or the realized value of the public information. With regard to the second element, agents trade-off the price vs. the likelihood of getting the good and they shade their bids as in the literature mentioned above. In other words, our model is a standard wallet game where we add a third wallet whose content is either observed by no bidder (common uncertainty) or by both bidders (public information).

It is crucial to note that the nature and the quantity of information that each agent possesses changes over rounds in a non-linear way. As we move from one round to the next, not only the support but the entire distribution function of the bidder's private information,  $F^r(\cdot)$ , changes. Because the distribution (and not just the realized values) affects the optimal bid, *it is not* possible to decompose the optimal bid in round  $r + 1$  into the optimal bid in round  $r$  plus a bid differential due to the revelation of  $x_{r+1}$ .

### 3 Experimental setting

We conducted 6 sessions with 8 or 10 subjects per session for a total of 52 subjects. The subjects were students at the California Institute of Technology who were recruited by email solicitation. All sessions were conducted at the Social Science Experimental Laboratory (SSEL). Interaction between subjects was computerized using an extension of the open source software package Multistage Games.<sup>2</sup> No subject participated in more than one session.

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<sup>2</sup>Documentation and instructions for downloading the software can be found at <http://multistage.ssel.caltech.edu>.

In each session, subjects made decisions over 15 paid matches, with each match being divided into 5 rounds. At the beginning of a match, subjects were randomly matched into pairs and randomly assigned a role as bidder  $A$  or bidder  $B$ . Pairs and roles remained fixed for the 5 rounds of a match. At the end of the match, subjects were randomly rematched into new pairs and they were reassigned new roles. All participants started the experiment with an endowment of 400 tokens.

The game closely followed the setting described in section 2. Subjects in a pair had to bid in a first price sealed bid auction for a good made of  $N = 6$  components. Each component  $i \in \{1, \dots, 6\}$  contained  $x_i$  tokens drawn from a uniform distribution in  $[0, 50]$  (so  $G(x_i) = x_i/50$ ) although, to simplify computations, we restricted  $x_i$  to integer values,  $\{0, 1, \dots, 50\}$ . The total value of the good was common to both bidders and equal to the sum of the six components. Visually, each component was represented by a box in the computer screen. The number of tokens inside each of the six boxes was drawn at the beginning of the match. Subjects could see the six boxes but not their content.

The match was then divided into 5 rounds. Round 1 corresponded to  $r = 1$  in the theory section. Subject  $A$  observed  $x_1$  (the content of box 1) and subject  $B$  observed  $x_6$  (the content of box 6). Neither subject observed  $x_2$  to  $x_5$  (the content of boxes 2 to 5). Given this information, both participants submitted a bid for *the entire good* of value  $V = \sum_{i=1}^6 x_i$ . Subjects could not see the bid of the other subject, instead they moved to round 2, which corresponded to  $r = 2$  in the theory section. Subjects  $A$  and  $B$  could now see the content of a second box ( $x_2$  for bidder  $A$  and  $x_5$  for bidder  $B$ ), and placed a new bid again for the entire good  $V$ . This process continued until round 5,  $r = 5$  in the theory section, where  $A$  observed  $x_1$  to  $x_5$  and  $B$  observed  $x_2$  to  $x_6$ . At the end of round 5,  $V$  and the five bids of each subject were displayed on the screen. One of the rounds was randomly selected by the computer, and subjects were paid according to the outcome of that round. More precisely, payoffs were computed according to the standard rules of a first price auction without reserve price: the highest bidder would get the item and pay his bid, while the lowest bidder would get nothing and pay nothing. A sample screenshot of the user interface in round  $r = 2$  is presented in Figure 1. It displays the subject's role, the current round, the stock of tokens, the content of the open boxes, a reminder of which boxes have been opened by the rival, and the subject's own bid(s) in the previous round(s) of that match.

Summing up, the only variable that changed between rounds was the amount of information each bidder had, which increased from  $r$  to  $r + 1$ . By contrast, the opponent,

role and total value of the item remained the same for the entire match. Naturally, it was crucial not to disclose the bids of the opponent between rounds since they contained information which could have been used as a signaling device, making the theoretical analysis substantially more intricate. We realize that this design is susceptible to anchoring effects due to the sequential revelation of information. Whether anchoring happened or not is an empirical question that the data analysis can answer (see section 4.3). We felt that this presentation was simple to understand and facilitated the computation of bids, since only one new box was revealed in each round.

The payoffs of each match were added or subtracted from the initial endowment. At any moment, the participants could not bid more than their current stock of tokens, resulting in a potential selection effect due to liquidity constraints. However, this constraint was never binding in the experiment. Indeed, the stock of tokens of all participants was always greater than 300, the maximum value of the item and therefore the maximum possible bid.<sup>3</sup>

At the beginning of each session, instructions were read by the experimenter (a sample copy of the instructions are in the Appendix). The experimenter explained the rules and how to operate the computer interface. After the instruction period, one practice match was conducted for which subjects received no payment followed by an interactive computerized comprehension quiz that all subjects had to answer correctly before proceeding to the paid rounds.<sup>4</sup> Then, the 52 subjects participated in 15 paid matches each of them divided into 5 rounds for a total of 75 bids per subject. Opponents, roles and values in all boxes were randomly reassigned at the beginning of each match and held constant between rounds of a match. In the end, subjects were paid, in cash, in private, their earnings, which were equal to their initial endowment plus the payoffs of all matches. The conversion rate was \$1.00 for 25 tokens, so goods were worth between \$0 and \$12. Sessions averaged one hour in length, and subjects earnings averaged \$27 including a \$5 show-up fee.

## 4 Aggregate analysis

The objective of this section is to test the predictions of Lemma 1 at the aggregate level. Table 1 presents the closed-form solutions of the bidding functions in all rounds for our

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<sup>3</sup>We constrained the bids to be between 0 and 300: the minimum and maximum possible values of the good before any information is revealed.

<sup>4</sup>Subjects who failed to answer the quiz correctly were coached individually until they could understand and answer all questions correctly.



particular case with 6 boxes and values drawn from a uniform distribution in  $[0, 50]$ .

## 4.1 Hypotheses

We first state the two main hypothesis we will be testing. Each corresponds to a prediction of Lemma 1.

**Hypothesis 1.** *The average bid is U-shaped across rounds while the average payoff is hump-shaped across rounds.*

Lemma 1 predicts that the expected NE bid  $E[b^r]$  is symmetric and U-shaped across rounds: decreasing over rounds when  $r \leq N/2$  and increasing over rounds when  $r \geq N/2$ .<sup>5</sup> This is natural because bid shading is increasing in the amount of private information only, given a Nash player always bids the realized amount of public information and the expected amount of common uncertainty. Furthermore, if both bidders reduce their bids, their ex-ante expected utility increases since the ex-ante expected value of the good is constant. This means that the expected utility is hump-shaped across rounds: increasing over rounds when  $r \leq N/2$  and decreasing over rounds when  $r \geq N/2$ . Even if we observe deviations from Nash in the data, we expect these qualitative properties of bids across rounds to hold.

**Hypothesis 2.** *Bids depend not only on the amount of information but also on the type of information (private vs. public).*

Lemma 1 predicts also that NE bids depend on both the quantity *and* the type of information. Sometimes, the last box revealed is a piece of private information (rounds 2 and 3) hence not observed by the other bidder. Other times, it is a piece of public information (rounds 4 and 5) observed by the other bidder as well. Again, even though we anticipate deviations from Nash, we still expect our subjects to treat these two types of information differently. In the next two sections we test these two predictions in our experimental data.

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<sup>5</sup>The result is true at the boundaries for any distribution ( $E[b^0] > E[b^1]$  and  $E[b^{N-1}] < E[b^N]$  for all  $G(x_i)$ ). We conjecture that it should hold for all  $r$  when  $x_i$  is in a certain class of distributions but we have been unable to determine the properties of that class. Related problems have been analyzed by Shaked and Shanthikumar (1994), Bagnoli and Bergstrom (2005) and Mares and Shor (2008). Note, however, that in our setting both the support and the probability distribution function change from round to round, hence we cannot use the results derived in these papers.

## 4.2 Aggregate bids and payoffs

We start with a general description of deviations in bids. Figure 2 shows the difference between actual bids and NE predictions in each round. For each observation, we compute the NE bid and subtract it from the corresponding observation. The line in the middle is the median of this statistic, whereas the top and bottom lines are the 75th and 25th percentiles. The notches are the 95% confidence interval for the median. We can make two main observations. First, deviations from NE predictions exhibit a hump shaped pattern across rounds: they increase from round 1 to round 3 and decrease from round 3 to round 5. Given private information increases from round 1 to round 3 and decreases afterwards, this result could in principle be explained by subjects falling prey of the winner’s curse. However, such conclusion is misleading. Indeed, the median deviations in rounds  $r$  and  $r + 1$  are significantly different from each other at the 95% level for all  $r$ , and they are all significantly different from zero. More precisely, there is underbidding in round 1 and overbidding in rounds 2 to 5. The winner’s curse hypothesis alone cannot justify this finding. Second, the dispersion in the data decreases over rounds. Therefore, it is inversely related to the total amount of information (which increases over time). More precisely, decreasing common uncertainty (from rounds 1 to 3) or increasing public information (from rounds 3 to 5) reduces bid heterogeneity. These two findings taken together suggest that deviations from NE predictions cannot be entirely attributed to the winner’s curse and are likely to be related to imperfect accounts of the different types of information. We will delay further discussions of this point to the next section.

These preliminary findings establish that bidding departs largely from NE predictions. Recall that Lemma 1 predicts that the average NE bid is U-shaped across rounds. To better understand the direction of the departures in our data, we construct Table 2 that displays the average actual bids per round (Data) and the NE predictions. For comparison, it also displays the average best response to the empirical distribution (BR). To construct this table, we compute the NE bid and the best response to the empirical distribution for each observation. Ideally, we would want to compute the empirical distribution of the bids for each possible value of private information and to calculate the expected gain of each possible bid for each possible value of private information. The best response to the empirical distribution would be the bid that maximizes the expected gain. However, this procedure would require a massive amount of data. Thus, we decided instead to divide the values of private information into 52 bins, each with 15 observations.<sup>6</sup> We used the

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<sup>6</sup>In order to have the same precision when estimating the empirical distribution in each bin, we decided

average private information in each bin as the value of the opponent’s private information in that bin. Then, we computed the best response to the empirical distribution using the method described above.

We can see from Table 2 that the average bid is increasing across rounds, instead of U-shaped. Subjects become more confident in their information and bid more aggressively as their total information (whether it is private or public) increases. Note, however, that the difference between the average bid and the average NE prediction is relatively small in percentage terms (between 1% and 13%). Interestingly, the BR is hump-shaped across rounds, which is the opposite pattern of the NE predictions. Typically, it is optimal to underbid significantly in rounds 1 and 5 (bidding the NE implies winning with higher probability but very little gain) and overbid in round 3 (to compensate for the rival’s over-bidding).<sup>7</sup> Subjects deviate more from the best response in late rounds, with a maximum difference of 17% in round 5.

We then perform a similar analysis of expected payoffs. Table 3 displays the average actual gains per round (Data), the NE predictions and the gains obtained from best responding to the empirical distribution (BR).<sup>8</sup> We can see that the average gain in our sample is decreasing over rounds (instead of hump-shaped). This results from the fact that actual bids are increasing over rounds. Despite the small reported differences in bids, the percentage difference in gains is significant. By underbidding in round 1, subjects increase their payoff by 19% whereas by overbidding in rounds 2 to 5 they decrease their payoff between 15% and 38%. Moreover if subjects best responded to the empirical distribution their gains could have increased by as much as 50% in some rounds. We summarize the findings of this section in the following result.

**Result 1.** *Hypothesis 1 is not supported by the data. Bids are increasing across rounds instead of U-shaped and gains are decreasing across rounds instead of hump-shaped. Over-bidding in rounds 2 to 5 implies losses of up to 38%.*

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to have the same number of observations per bin. This implies that bins have different lengths.

<sup>7</sup>Round 5 is particularly interesting. Although the mean empirical bid coincides with the NE, the BR is still to underbid significantly. This is due to the *dispersion* in bids: by underbidding, subjects decrease the probability of winning the item but increase substantially the net gain in case of success.

<sup>8</sup>Remember that we only paid subjects for one randomly drawn round per match. However, when we mention ‘gain’ we refer to the profits subjects would have made had they been paid for that round. Notice that the loser of the auction makes zero profits whereas the winner can make positive or negative profits. Also, NE gains were computed as if all subjects in the population were bidding the NE.

### 4.3 Aggregate bids and types of information

The previous section described the general patterns of bids across rounds and established the direction of departures from NE predictions. The objective of this section is to test the second implication of Lemma 1, namely that different types of information should be accounted for differently. This will allow us to relate departures to the treatment of the different types of information, which is the key novelty of our design.

The first natural question is whether subjects realize that the different types of information should be treated differently. For instance, in rounds 2 and 3 the last box revealed is a piece of private information whereas in rounds 4 and 5, the last box revealed is a piece of public information. Therefore, NE predicts that the change in bids from round 1 to 2 and from round 2 to 3 should be qualitatively different from the change in bids from round 3 to 4 and from round 4 to 5 (see Lemma 1, where  $b^{r+1}(X_j^{r+1}) - b^r(X_j^r)$  depends on  $r$ ). To see whether this is the case, we construct Table 4.

In the absence of data constraints, we would like to analyze the change in bids as a function of the value of the boxes previously opened and the value of the new box, so as to better understand how subjects react to new information given their previous information. Due to the limited amount of data, we have to use an aggregate statistics. We organize the two variables somewhat arbitrarily into high values and low values. High values (H) correspond to cases in which the sum of the values in the boxes already opened (respectively the value in the new box) is above the expected amount of those boxes (respectively that box). Similarly, low values (L) correspond to cases in which the sum of the values in the boxes already opened (respectively the value in the new box) is below the expected amount of those boxes (respectively that box). We construct four groups 'H to H', 'H to L', 'L to H' and 'L to L' where the first letter refers to the value of the sum of opened boxes and the second to the value of the new box. For instance, 'H to L' means that the sum of the opened boxes is above expectation and the value in the new box is below expectation. For each of these four groups we calculate the average change in bids. Each column in Table 4 contains the average change in bids between rounds. For example,  $r1 - r2$  refers to the change between rounds 1 and 2. For each group we report the NE predictions and the empirical data. Finally, the last two columns display a normal and non-parametric test for the overall difference in means across rounds.

According to NE predictions, change in bids should be driven mostly by the new information. Most importantly, changes in bids from  $r1$  to  $r2$  and  $r2$  to  $r3$  should be

vastly different from changes in bids from  $r3$  to  $r4$  and  $r4$  to  $r5$ . Indeed, in the early rounds 2 and 3, the new box contains private information, which should be shaded. By contrast, in the late rounds 4 and 5, the new box contains public information, which should be entirely reflected in the bids. Therefore, when the new box is above average, the increase in bids should be more dramatic in late rounds than in early rounds. Conversely, when the new box is below average, the decrease in bids should be higher in early rounds than in late rounds.

We see that the actual average changes in bids do not exhibit these patterns. Subjects do change their bids when new information is disclosed (that is, there is no strong evidence of anchoring effects). However, the magnitudes of the average changes in bids across rounds are very different from NE predictions, and we do not observe the predicted marked difference between early and late rounds. Even though the difference across rounds is significant at the 5% level in two groups and at the 10% level in three groups according to the non-parametric test, the patterns are erratic and the magnitudes are small. For example, in the ‘H to H’ group NE predicts the increase in bids to be almost 6 times higher in  $r3 - r4$  than in  $r2 - r3$ , whereas in the data the increase is about 3%. Overall, at the aggregate level, subjects react to the amount of new information but not to its type. This finding is summarized in the following result.

**Result 2.** *Hypothesis 2 is not supported by the data. The reaction of subjects to new information is largely independent of the type of information revealed (public vs. private).*

## 5 Cluster Analysis

The aggregate analysis establishes that bids per round depart from NE predictions, and that those deviations are related to an incorrect treatment of information. However, Figure 2 also reveals a large dispersion in deviations across rounds. It is therefore plausible that different groups of subjects behave differently in each round. To study this question, we search for trends at a disaggregate level. One possible approach is to do a subject-by-subject analysis (as in Costa-Gomes et al. (2001) for example). Even though this is in theory the most informative strategy, the reduced number of observations for each individual in each round would prevent us from making confident assessments. We therefore take an alternative approach, which is to search for clusters of subjects (as in Camerer and Ho (1999) and Brocas et al. (2013)). This is an intermediate approach, as the aggregate approach implicitly assumes a single cluster is of interest while the subject-specific

approach requires each subject to be in a singleton cluster. One advantage of the method is to provide an implicit measure of how well these two extreme cases capture the observed behavior.

## 5.1 Method

We do not know a priori the behavioral types of our subjects nor the shape of their actual bidding function. We cannot therefore assume confidently that their behavior is generated by some predetermined model. An agnostic way to reveal types is to search for patterns of deviations from NE across rounds, and to group subjects according to those patterns. A follow-up analysis of each cluster can then reveal the causes of the associated patterns and help identify types. This method, which has been successfully used in Brocas et al. (2013), will also be adopted here.

To find the clusters, we use for each of the 52 subjects the average deviation from NE in each round. Each subject is thus represented by five averages (rounds 1 to 5). There is a wide array of heuristic clustering methods that are commonly used but they usually require the number of clusters and the clustering criterion to be set ex-ante rather than endogenously optimized. By contrast, mixture models treat each cluster as a component probability distribution. Thus, the choice between different numbers of clusters and different models can be made using Bayesian statistical methods (Fraley and Raftery, 2002). Note that popular heuristic approaches such as ‘k means clustering’ are equivalent to mixture models where a particular covariance structure is imposed.

Since we do not want to presuppose a particular structure, we implement a model-based analysis with an endogenously optimized number of clusters. We consider a maximum of ten clusters and assume a diagonal covariance matrix. This implies zero correlation between the dimensions and no restriction on the variance. We first fix the number of clusters from 1 to 10, and for each model we estimate the covariance matrix as well as the clustering that maximizes the likelihood function. We use random clustering as an initial guess.<sup>9</sup> Overall, for any possible number of clusters we obtain a clustering and a covariance matrix for each cluster, and we compute the corresponding Bayesian Information Criterion (BIC). Given this information, the optimal number of clusters is the one for which BIC is maximized. For our data, the BIC is maximized for 6 clusters with diagonal covariance

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<sup>9</sup>We ran the model several times and the results were consistent. We use the EM algorithm for maximum likelihood estimation of multivariate mixture models. See Fraley and Raftery (2002) for a description of the clustering method.

matrix. These clusters, labeled 1 to 6, contain 11, 13, 6, 6, 9 and 7 subjects respectively. Notice that clusters are of different but comparable size, although this does not necessarily need to be the case.

## 5.2 Bidding behavior by cluster

In order to determine the properties of our clusters, we perform the same aggregate analysis as in section 4 for each cluster separately. Figure 3 is the equivalent of Figure 2 for each cluster, that is, the average deviation from NE predictions per round. Table 5 reports the average gains in each cluster. Finally, Table 6 presents for each cluster the equivalent of Table 4, that is, the change in bids conditional on the type of information revealed.

Clusters 1 and 2 are closest to the theory. On average, they bid close to NE (Figure 3). They also react differently to the different types of information. This reaction shares the qualitative properties of the theory. Yet, significant quantitative departures can be noted: subjects do not react nearly as much as theory predicts and they do not react uniformly over all four groups (Table 6). A few other differences relative to NE can be noted. Subjects in cluster 1 slightly overbid in every round while subjects in cluster 2 slightly underbid in every round. Interestingly, even though they bid close to Nash, these subjects do not obtain the highest payoffs. This is true because they do not best respond to the empirical distribution (recall that it is optimal to underbid significantly in rounds 1 and 5).

Cluster 3 is composed of a relatively heterogeneous group of subjects who underbid significantly in every round. Subjects change their bids across rounds but in an unpredictable way (often in the direction opposite from that predicted by theory), suggesting that they do not realize the presence of different types of information. They obtain relatively high payoffs because their underbidding strategy coincides often with the best response to the empirical distribution.

Subjects in cluster 4 overbid significantly in every round and are very homogeneous. The average change in bids of this cluster is similar in all rounds, which means they treat private and public information in the same way. Their average gains are lowest given their substantial overbidding and the optimality of underbidding.

Cluster 5 is an interesting group of subjects. On the one hand, they bid as close to Nash as clusters 1 and 2, and with no systematic deviation (there is slight underbidding in round 1 and slight overbidding in round 3). In that dimension they look closest to

rational players. On the other hand, they do not have a discernible pattern regarding the change in bids over rounds, which casts doubts about their understanding of the different types of information. They obtain the highest gains both overall and per round since their bidding strategy is closest to best response to the empirical distribution.<sup>10</sup>

Last, subjects in cluster 6 underbid substantially (most notably in round 1) and are extremely heterogeneous. They react tremendously to new information but very similarly if it is private or public. Their bids are significantly lower and more dispersed than those of cluster 3. Subjects in this cluster lose the auction most of the time and therefore obtain small payoffs. These findings are summarized below.

**Result 3.** *Subjects in different clusters have different departures from NE, reaction to information, and degrees of heterogeneity which can be summarized as:*

Cluster	Difference from NE	Type of info. matters	Heterogeneity
1	<i>Small / Overbid</i>	<i>Yes</i>	<i>Small</i>
2	<i>Small / Underbid</i>	<i>Yes</i>	<i>Small</i>
3	<i>Large / Underbid</i>	<i>No</i>	<i>Large</i>
4	<i>Large / Overbid</i>	<i>No</i>	<i>Very small</i>
5	<i>Small / Round dependent</i>	<i>No</i>	<i>Small</i>
6	<i>Very large / Underbid</i>	<i>No</i>	<i>Large</i>

As in previous studies (e.g., Crawford and Iriberri (2007), Brocas et al. (2013)), we find clusters of subjects characterized by homogeneity within groups and heterogeneity across groups. The heterogeneity across clusters suggests that subjects may have different cognitive abilities. This is most apparent for clusters 1 and 2 on one end and 4 and 6 on the other. Subjects who strongly underbid (cluster 6) or systematically overbid (cluster 4) are likely to use simple heuristics. They fail to realize the link between bids and information and end up collecting the smallest payoffs. Subjects who play relatively close to NE (clusters 1 and 2) look sophisticated enough to approximate equilibrium behavior for all types of information but, at the same time, they overestimate the ability of their opponents to play Nash. As a consequence, they do not obtain the highest gains. The reasoning made by clusters 3 and 5, who obtain the highest payoffs, is more difficult to grasp. The strong underbidding and unpredictable changes of bids across rounds by cluster 3 suggests they may be using a ‘lucky’ heuristic that turns out to work well. As

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<sup>10</sup>It would be interesting to investigate whether these subjects learn over time to best respond to the empirical behavior of the population. We could not perform this analysis due to the reduced number of observations.



for cluster 5, their ability to bid close to NE and to depart from it at the correct time is intriguing. Perhaps they do realize the limitations of their opponents and take advantage of this knowledge.

Last, the comparison between Figures 2 and 3 is interesting. Indeed, even though there is a lot of dispersion in the data at the aggregate level, there is relatively little dispersion within most clusters in each round. This suggests that there is little learning taking place in our auction. Each type of subject acts in a similar way in all trials of the same round. Moreover, each type of subject adapts his behavior from round to round in a consistent way. Except for the clueless and volatile cluster 6 (13% of the population), most subjects seem to be selecting a strategy in each round rather than leaving their decisions to chance. Overall, there is a large heterogeneity in strategies between individuals but not much heterogeneity within individuals and across matches.

### 5.3 Analysis of bidding strategies for clusters 1 and 2

One important finding of the cluster analysis is that approximately half of the subjects (the 28 subjects in clusters 3, 4, 5 and 6) do not treat private and public information differently. The other half (the 24 subjects in clusters 1 and 2) do realize to some extent that the type of information matters. In this section, we study in more detail the relationship between the empirical and NE bidding functions of these subjects. Given these subjects bid reasonably close to equilibrium, we will estimate a bidding function that keeps the main properties of the NE bidding function but allows for slight departures. This is explained more carefully below.

The NE bidding function is a linear function of public information (with slope 1) and a polynomial function of private information. In rounds 1 and 5, the polynomial is of degree 1, whereas in rounds 2, 3 and 4 it is the ratio of two higher order polynomials (see Table 1). We consider a polynomial approximation of our bidding function. Namely, for each round, we compute a cubic approximation in which we set the following relationship between the NE bid,  $b^r$ , and the various types of information:

$$b^r = \alpha_0^r + \alpha_1^r Priv^r + \alpha_2^r (Priv^r)^2 + \alpha_3^r (Priv^r)^3 + \alpha_4^r Pub^r + \eta_{Priv^r}$$

In this equation, superscript  $r$  denotes the round,  $Priv$  is the variable of private information and  $Pub$  is the variable of public information. The constant term  $\alpha_0^r$  is the coefficient of common uncertainty, that is, the expected number of tokens in the boxes nobody observes. These are 100 in round 1, 50 in round 2 and 0 in the remaining rounds under the

exact polynomial expression for  $Priv$ . The coefficient of public information,  $\alpha_4^r$ , is relevant only in rounds 4 and 5 ( $\alpha_4^1 = \alpha_4^2 = \alpha_4^3 = 0$ ). It is equal to 1 since both bidders compete à la Bertrand. Also,  $\alpha_1^r$ ,  $\alpha_2^r$  and  $\alpha_3^r$  are the coefficients for the cubic approximation of private information. Finally,  $\eta_{Priv^r}$  is the error term for each level of private information. It is different from zero in rounds 2, 3 and 4, where the cubic formulation of private information is only an approximation. The  $\alpha^r$ -coefficients are reported in Table 7 for each round.<sup>11</sup>

If subjects play according to NE theory, they should produce bids consistent with  $b^r$  and parameters  $\alpha_k^r$  ( $k = 0, 1, 2, 3, 4$ ). Given we know they do not exactly bid according to the model, we will allow them to bid according to a modified bidding function that preserves the overall NE structure but allows for different parameters. We will then fit this modified bidding function for the 24 subjects in clusters 1 and 2 to understand and compare the parameters we obtain with  $\alpha_k^r$ . More precisely, we consider the following regression:

$$b_o = \beta_0 + \sum_{r=1}^4 D_r [\beta_0^r + \beta_1^r Priv_o^r + \beta_2^r (Priv_o^r)^2 + \beta_3^r (Priv_o^r)^3 + \beta_4^r Pub_o^r] + \varepsilon_o$$

where superscript  $r$  denotes the round, subscript  $o$  denotes the observation and  $D_r$  is a round specific dummy. We see the similarity with the NE approximation: the  $\alpha_k^r$ -coefficients have been replaced by  $\beta_k^r$ -coefficients with the same interpretation. We compute the coefficients of the Feasible Generalized Least Squares (FGLS) Random Effects regression for each of the variables and the t-test to check if the coefficients are significantly different from those predicted by NE ( $\alpha_r^k$  parameters above).<sup>12</sup> We also perform a global significance F-test to inspect if, overall, the data bidding function is different from the NE bidding function. Figure 4 and Table 7 display the results of this exercise. Our main findings are the following.

First, compared to NE, the proportion of the bid driven by common uncertainty is smaller in round 1, similar in round 2, and higher in the other rounds. Overall, deviations from NE regarding common uncertainty are hump-shaped, and largest in round 3. This result follows from a comparison between the estimated  $\beta_0^r$ -coefficients and the predicted

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<sup>11</sup>We opted for a cubic rather than quadratic approximation, because the latter performs badly for extreme values of private information. For example, in round 3, the quadratic approximation of NE has a constant term of -20 instead of the theoretical prediction of 0.

<sup>12</sup>The FGLS estimator makes use of the panel data structure to get more precise coefficients. We also performed the Hausman test and we could reject the presence of unobserved fixed effects. The variance estimator is clustered by subject.

NE  $\alpha_0^r$ -coefficients. Second, subjects react less to public information than what theory predicts, although the differences are not large. Indeed,  $\beta_4$  is close to but smaller than the corresponding NE prediction in the relevant rounds 4 and 5.<sup>13</sup> This results in overbidding for low values of public information and underbidding for high values of public information. Notice that risk aversion would impact both private information and common uncertainty, but not public information.<sup>14</sup> Third, a close inspection of Figure 4 suggests that subjects react less to private information than what NE predicts in round 1, and therefore underbid for all values of private information in that round. In rounds 2 and 4, subjects overbid for low values and underbid for high values of private information whereas in round 3 there is consistent overbidding. We summarize the findings in the following result.

**Result 4.** *The subjects who play close to NE exhibit nonetheless some departures with respect to all three types of information: (i) they under-react to common uncertainty in early rounds and over-react in late rounds; (ii) they consistently under-react to public information; and (iii) they typically overbid when their private information is large.*

## 6 Conclusions

This paper incorporates two novel features that facilitate the study of bidding behavior in common value auctions. First, it divides the good into three elements: those known by both bidders, those known by one bidder, and those known by no bidder. Second, it varies the relative importance of each element holding everything else constant. The paper replicates the overbidding tendency documented in previous research (Kagel and Levin (1986, 2008)). More importantly, we show that only half of the subjects grasp the idea that different types of information imply different adjustments of bids. The other half treat new information in the same way, independently of the type of information being revealed. Those who understand that the information structure matters still under- or over-react to information.

We believe this is an important point with relevance in many experimental designs. For instance, the auction studied in this paper has a signal extraction problem which is similar to other games with common values and private information: informational

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<sup>13</sup>Bids that are lower than NE predictions vis-à-vis public information is equivalent to prices that are higher than NE predictions in Bertrand competition games, a result found in Abbink and Brandts (2008).

<sup>14</sup>Holt and Sherman (2000) show in a specific setting that risk aversion does not affect the bidding function. However, for general distributions two effects operate in opposite directions for risk averse bidders: an uncertain prize is valued less (implying lower bids) but in order to reduce the uncertainty associated to losing the auction, the bidder prefers to bid more.

cascades, information aggregation through voting, and jury verdicts just to name a few. Our result suggests that for these settings, one should pay close attention to the way in which information is presented. Different models may have qualitatively the same theoretical predictions. In practice, however, the behavior of subjects may be affected by the presence and relative importance of peripheral elements such as public information and common uncertainty.

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## 7 Appendix

### 7.1 Proof of Lemma 1

We restrict the attention to monotonic bidding strategies that are differentiable. Assume that bidder  $B$  bids according to such a function and denote it by  $b^r(X_B)$ .

Let  $r < N/2$ . The expected utility of bidder  $A$  when he bids  $b_A^r$  is  $U_A^r = \Pr(b_A^r \geq b^r(X_B^r)) (X_A^r + E[X_\emptyset^r] + E[X_B^r | b_A^r \geq b^r(X_B^r)] - b_A^r)$ . Using the distribution of  $X_A^r$ , it can be rewritten as:

$$U_A^r = (X_A^r + E[X_\emptyset^r] - b_A^r) F^r(b^{r^{-1}}(b_A^r)) + \int_{\underline{X}^r}^{b^{r^{-1}}(b_A^r)} X_B^r f^r(X_B^r) dX_B^r \quad (1)$$

Maximizing  $U_A$  with respect to  $b_A^r$  and imposing the symmetry condition  $b_A^r = b^r$ , we get the following first-order condition:

$$(2X_A^r + E[X_\emptyset^r]) f^r(X_A^r) = F^r(X_A^r) b^{r'}(X_A^r) + b^r(X_A^r) f^r(X_A^r)$$

Integrating both sides yields the result. The ex-ante expected bid when  $r < N/2$  is:

$$E[b^r] = \int_{\underline{X}^r}^{\bar{X}^r} \left( E[X_\emptyset^r] + 2 \left( X_A^r - \frac{\int_{\underline{X}^r}^{X_A^r} F^r(s) ds}{F^r(X_A^r)} \right) \right) f^r(X_A^r) dX_A^r$$

Integrating the last term by parts, we get:

$$\begin{aligned} E[b^r] &= E[X_\emptyset^r] + 2E[X_A^r] + 2 \int_{\underline{X}^r}^{\bar{X}^r} \log(F^r(X_A^r)) F^r(X_A^r) dX_A^r \\ \Leftrightarrow E[b^r] &= E[V] + 2 \int_{\underline{X}^r}^{\bar{X}^r} \log(F^r(X_A^r)) F^r(X_A^r) dX_A^r \end{aligned}$$

The ex-ante expected utility of bidder  $A$  is  $E[U_A^r] = \int_{\underline{X}^r}^{\bar{X}^r} U_A^r(X_A^r) f^r(X_A^r) dX_A^r$  and at equilibrium  $U_A^r(X_A^r) = \int_{\underline{X}^r}^{X_A^r} F^r(s) ds$ . Integrating by parts, we get

$$E[U_A^r] = \int_{\underline{X}^r}^{\bar{X}^r} F^r(X_A^r) (1 - F^r(X_A^r)) dX_A^r$$

When  $r = N/2$  there is no common uncertainty so we just need to remove the term  $E[X_\emptyset^r]$ . When  $r > N/2$ , We have a family of functions parameterized by  $X_{\text{pub}}^r$ . Therefore, we can substitute  $E[X_\emptyset^r]$  by  $X_{\text{pub}}^r$  and follow the same procedure to get the result.  $\square$

## 7.2 Figures

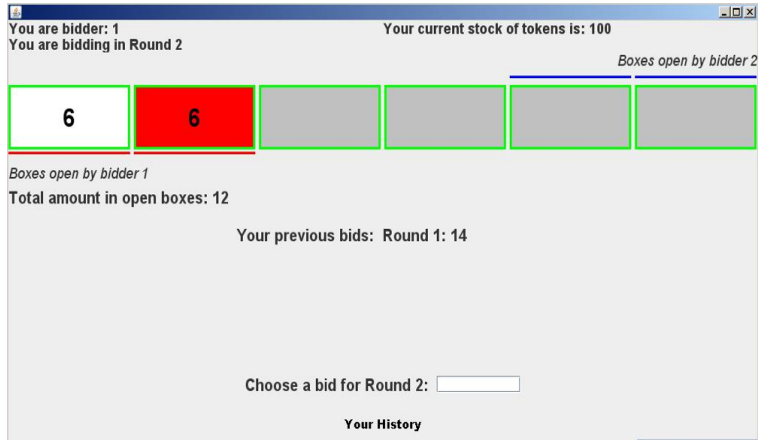


Figure 1: Sample screenshot of user interface

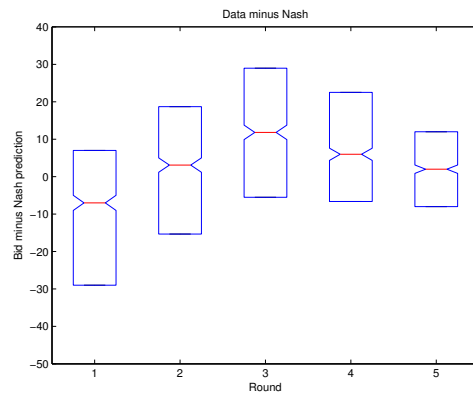


Figure 2: Deviations from NE

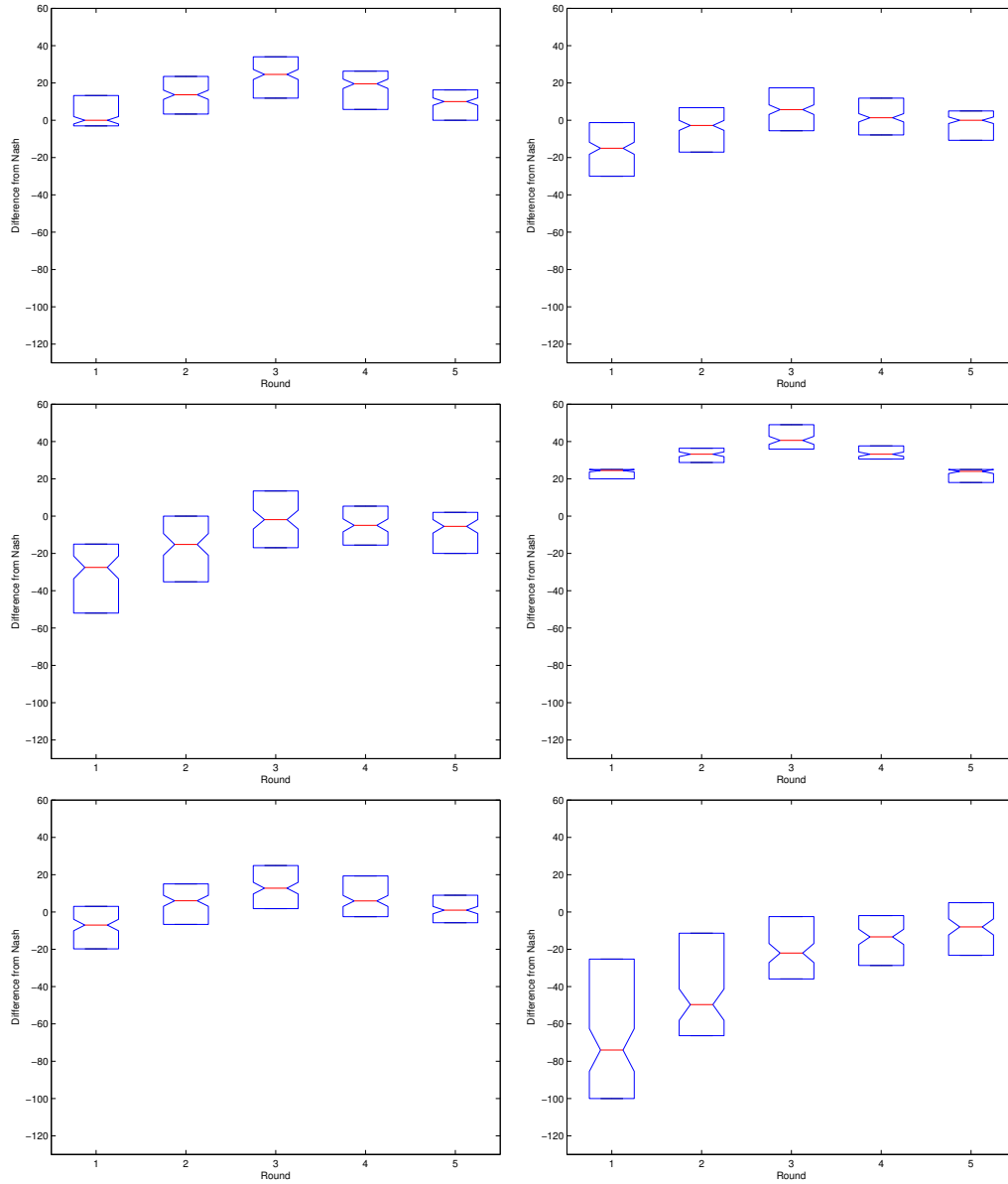


Figure 3: Deviations from NE per cluster



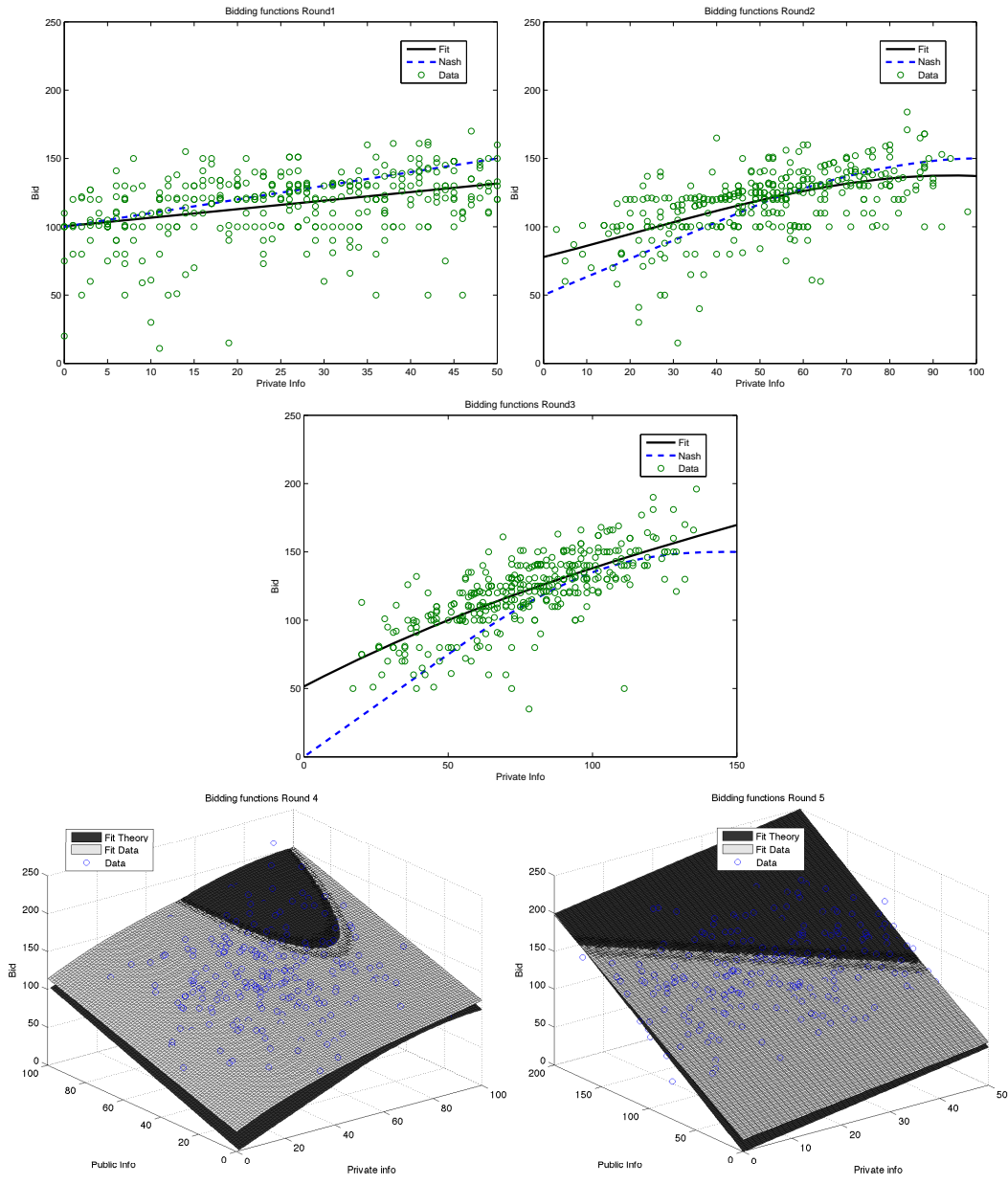


Figure 4: NE and FGLS regression for clusters 1 and 2

### 7.3 Tables

Table 1: Bidding functions per round

Round	Nash Equilibrium bid
1	$100 + 2 \left( X_j^1 - X_j^1/2 \right)$
2	$50 + 2 \left( X_j^2 - X_j^2/3 \right)$ , if $X_j^2 \leq 50$ $50 + 2 \left( X_j^2 - \frac{125000/3 - 2450 - X_j^2 + (X_j^2)^2 - (X_j^2)^3/6}{2400 + 2X_j^2 - (X_j^2)^2/2} \right)$ , if $X_j^2 > 50$
3	$2 \left( X_j^3 - X_j^3/4 \right)$ , if $X_j^3 \leq 50$ $2 \left( X_j^3 - \frac{720600 + X_j^3/2 - 3(X_j^3)^2/4 + (X_j^3)^3/2 - (X_j^3)^4/12}{58825 - 3X_j^3/2 + 3(X_j^3)^2/2 - (X_j^3)^3/3} \right)$ , if $50 < X_j^3 \leq 100$ $2 \left( X_j^3 - \frac{93.75 - 7X_j^3/2 + 9(X_j^3)^2/4 - (X_j^3)^3/2 + (X_j^3)^4/24}{-175 + 9X_j^3/2 - 3(X_j^3)^2/2 + (X_j^3)^3/6} \right)$ , if $X_j^3 > 100$
4	$X_P + 2 \left( X_j^2 - X_j^2/3 \right)$ , if $X_j^2 \leq 50$ $X_P + 2 \left( X_j^2 - \frac{125000/3 - 2450 - X_j^2 + (X_j^2)^2 - (X_j^2)^3/6}{2400 + 2X_j^2 - (X_j^2)^2/2} \right)$ , if $X_j^2 > 50$
5	$X_P + 2 \left( X_j^1 - X_j^1/2 \right)$

Table 2: Average bids

Round	1	2	3	4	5
Mean Data	109.41 (1.28)	113.52 (1.22)	116.70 (1.22)	120.90 (1.25)	127.40 (1.40)
Mean NE	125.10*** (0.51)	115.39 (0.81)	106.85*** (1.06)	115.51*** (1.12)	128.16 (1.20)
Mean BR	109.75 (0.51)	113.69 (0.61)	113.03*** (0.78)	111.79*** (0.80)	108.86*** (0.98)

Standard errors in parenthesis

\*, \*\*, \*\*\*: Significantly different from data at 90%, 95% and 98% confidence level (t-test)

Table 3: Average gains

Round	1	2	3	4	5
Mean Data	12.33 (1.09)	10.69 (0.96)	9.19 (0.84)	8.38 (0.67)	6.27 (0.50)
Mean NE	10.03** (0.90)	12.52* (0.82)	14.95*** (0.77)	12.46*** (0.61)	8.50*** (0.43)
Mean BR	17.40*** (1.10)	15.87*** (0.96)	12.76*** (0.81)	10.82*** (0.69)	6.04 (0.51)

Standard errors in parenthesis

\*, \*\*, \*\*\*: Significantly different from data at 90%, 95% and 98% confidence level (t-test)

Table 4: Average change in bids over rounds

		$r1 - r2$	$r2 - r3$	$r3 - r4$	$r4 - r5$	p-val nor <sup>†</sup>	p-val non <sup>††</sup>
L to L	NE	-28.64	-26.47	-2.45	0.34	0.00	0.00
	Data	-7.29	-9.63	-9.44	-8.45	0.54	0.31
L to H	NE	3.96	9.23	24.32	25.96	0.00	0.00
	Data	13.21	13.27	15.92	17.49	0.02	0.00
H to L	NE	-21.40	-19.00	-5.56	-2.10	0.00	0.00
	Data	-5.95	-5.79	-4.98	-2.41	0.11	0.06
H to H	NE	2.50	3.36	19.63	22.67	0.00	0.00
	Data	13.29	15.26	15.69	16.03	0.33	0.01

†: ANOVA test. ††: Kruskal-Wallis test

Table 5: Average gains by cluster

Cluster	1	2	3	4	5	6
Mean	9.56	10.12	10.46	4.73	12.31	6.97
St. Dev.	(5.39)	(6.38)	(5.55)	(4.57)	(6.35)	(5.77)

Table 6: Average change in bids over rounds per cluster

			$r1 - r2$	$r2 - r3$	$r3 - r4$	$r4 - r5$	p-val nor <sup>†</sup>	p-val non <sup>††</sup>
Cluster 1	L to L	NE	-29.46	-24.07	-3.15	-2.13	0.00	0.00
		Data	-9.55	-9.02	-9.95	-15.15	0.35	0.31
	L to H	NE	2.40	7.15	23.63	26.36	0.00	0.00
		Data	8.56	8.70	11.83	12.74	0.11	0.09
	H to L	NE	-21.13	-19.55	-5.67	-1.03	0.00	0.00
		Data	-5.69	-8.38	-5.97	-3.24	0.27	0.31
	H to H	NE	2.01	4.30	20.18	22.82	0.00	0.00
		Data	10.88	12.36	15.48	16.48	0.08	0.03
Cluster 2	L to L	NE	-26.63	-27.04	-2.09	1.47	0.00	0.00
		Data	-5.88	-9.32	-7.75	-6.56	0.64	0.45
	L to H	NE	3.86	10.04	24.12	27.13	0.00	0.00
		Data	11.42	13.12	17.58	22.31	0.00	0.00
	H to L	NE	-21.39	-19.23	-5.18	-3.22	0.00	0.00
		Data	-8.45	-8.22	-3.76	-3.43	0.21	0.04
	H to H	NE	2.86	3.50	20.80	23.89	0.00	0.00
		Data	11.55	15.00	14.68	17.21	0.20	0.05
Cluster 3	L to L	NE	-28.47	-27.44	-3.37	-0.15	0.00	0.00
		Data	-2.60	-5.95	-9.48	-7.00	0.62	0.39
	L to H	NE	4.63	9.45	25.89	25.46	0.00	0.00
		Data	10.15	16.19	7.44	14.71	0.36	0.61
	H to L	NE	-20.65	-23.49	-6.78	-0.22	0.00	0.00
		Data	-6.96	-0.78	-9.29	3.00	0.03	0.08
	H to H	NE	3.26	3.23	18.14	24.39	0.00	0.00
		Data	15.46	17.08	12.96	18.17	0.74	0.55
Cluster 4	L to L	NE	-27.03	-25.18	-5.13	1.80	0.00	0.00
		Data	-14.52	-12.29	-17.82	-12.57	0.37	0.47
	L to H	NE	4.67	7.90	26.78	26.29	0.00	0.00
		Data	12.77	7.94	15.14	12.42	0.30	0.20
	H to L	NE	-22.33	-18.32	-4.98	-6.38	0.00	0.00
		Data	-7.68	-12.48	-5.95	-14.92	0.08	0.10
	H to H	NE	3.73	1.39	20.21	20.52	0.00	0.00
		Data	14.93	15.00	14.00	11.84	0.74	0.61
Cluster 5	L to L	NE	-31.22	-28.98	-0.13	-0.58	0.00	0.00
		Data	-8.89	-13.55	-12.85	-10.17	0.75	0.62
	L to H	NE	5.22	8.70	23.38	24.30	0.00	0.00
		Data	19.66	10.39	20.25	17.25	0.23	0.19
	H to L	NE	-19.66	-15.93	-5.40	-0.76	0.00	0.00
		Data	-4.81	-4.55	-9.61	-5.85	0.37	0.46
	H to H	NE	2.41	2.23	18.48	22.21	0.00	0.00
		Data	11.14	8.64	11.36	15.77	0.06	0.19
Cluster 6	L to L	NE	-29.28	-27.25	-1.16	1.34	0.00	0.00
		Data	-1.76	-5.48	-0.04	-3.66	0.89	0.81
	L to H	NE	3.51	11.54	22.76	24.89	0.00	0.00
		Data	17.41	21.88	23.19	30.40	0.29	0.57
	H to L	NE	-23.64	-19.43	-5.81	-2.69	0.00	0.00
		Data	0.33	3.83	2.18	8.32	0.60	0.12
	H to H	NE	1.55	4.23	19.16	21.84	0.00	0.00
		Data	20.90	28.11	30.22	16.29	0.28	0.47

†: ANOVA test. ††: Kruskal-Wallis test

Table 7: NE and FGLS regression per round for clusters 1 and 2

	Round 1		Round 2		Round 3		Round 4		Round 5	
	NE	Data	NE	Data	NE	Data	NE	Data	NE	Data
Priv	1	0.48*** (0.008)	1.18	2.33*** (0.02)	1.42	1.99*** (0.01)	1.18	1.80*** (0.009)	1	1.17*** (0.007)
(Priv) <sup>2</sup>	N/A	-0.01	0.007 (0.00)	-0.03*** (0.00)	0.004	-0.01*** (0.00)	0.007	-0.015*** (0.00)	N/A	-0.006 (0.00)
(Priv) <sup>3</sup>	N/A	N/A	-0.0001	0.0001*** (0.00)	-0.0001	0.0000*** (0.00)	-0.0001	0.0000*** (0.00)	N/A	N/A
Pub	N/A	N/A	N/A	N/A	N/A	N/A	1	0.88*** (0.001)	1	0.91*** (0.000)
Constant	100	85.19* (0.19)	53.00 <sup>†</sup>	53.00 (0.36)	-0.05 <sup>†</sup>	25.60*** (0.34)	0.59 <sup>†</sup>	17.56*** (0.20)	0	9.2*** (0.14)
F-test		5.8***								
Adjusted $R^2$		0.38								

Standard errors in parenthesis

\*, \*\*, \*\*\*: Significantly different from NE at 90%, 95% and 98% confidence level (t-test)

†: These values are not 50, 0 and 0 respectively because we are using a polynomial approximation of NE

## 7.4 Sample Instructions

This is an experiment in group decision making, and you will be paid for your participation in cash at the end of the experiment. Different participants may earn different amounts. What you earn depends partly on your decisions, partly on the decisions of others, and partly on chance. The entire experiment will take place through computer terminals, and all interaction between participants will take place through the computers. It is important that you not talk or in any way try to communicate with other participants during the experiments.

We will start with a brief instruction period. During the instruction period, you will be given a complete description of the experiment and will be shown how to use the computers. You must take a quiz after the instruction period. So it is important that you listen carefully. If you have any questions during the instruction period, raise your hand and your question will be answered so everyone can hear. If any difficulties arise after the experiment has begun, raise your hand, and an experimenter will come and assist you.

At the end of the session, you will be paid the sum of what you have earned in all matches, plus the show-up fee of \$5. Everyone will be paid in private and you are under no obligation to tell others how much you earned.

Your earnings during the experiment are denominated in tokens. You start the experiment with an endowment of 400 tokens. Depending on your decisions, you can earn more tokens or lose some tokens. At the end of the experiment, we will count the number of tokens you have and you will be paid \$1.00 for every 25 tokens. So you start the experiment with an endowment of 16 dollars.

The experiment will consist of 15 matches. In each match, you will be paired with one of the other participants in the experiment. Since there are 12 participants in today's session, there will be 6 pairs in each match. You are not told the identity of the participant you are matched with. Your payoff depends only on your decisions, the decisions of the one participant you are matched with and on chance. What happens in the other pairs has no effect on your payoff and vice versa. Your decisions are not revealed to participants in the other pairs.

We will now explain how each match proceeds. At the beginning of the match, the computer pairs you with another participant. Next, the computer randomly assigns with equal probability a role to each member as "bidder 1" or "bidder 2". In each match, each member of the pair will be asked to bid for one item. The following screenshot illustrates how the value of the item is calculated.

[SCREEN 1]

You will never see such a screen, but it is useful to understand the game. In this screen, there are 6 boxes. Each box contains a number of tokens which is equally likely to be any integer from 0 to 50. The number of tokens in one box is independent of the number of tokens in the other boxes. To be more precise, before each match begins the computer selects, with equal chance, a number of tokens between 0 and 50 for the first box; then it selects, with equal chance, a number of tokens between 0 and 50 for the second box, and so on until the 6th box. The value of the item is the sum of the tokens in all six boxes. In this example, the value is 96. Both participants will bid for this value. Note that given the number of tokens in each box varies from 0 to 50, the value of the item is always at least 0 and at most 300. The number of tokens in each box and the corresponding value of the item remain the same during the entire match. However, this information will not be displayed to you all at once. Instead, the information will be revealed sequentially.

More precisely, each match is divided into 5 rounds. Participants keep the same role and pair for all the rounds of the match. In each round of the match, there is one auction for the whole item. We now explain how the auction in each round works.

Round 1

[SCREEN 2]

Bidder 1 sees a screen similar to the upper part of the slide. Bidder 2 sees a screen similar to the lower part of the slide. Both bidders see all 6 boxes but they do not see what is inside all of them. More precisely, bidder 1 sees the number of tokens inside the first box starting from the left, the underscored

box. Bidder 2 sees the number of tokens inside the first box starting from the right, the overscored box. Note that knowing the number of tokens in one box does not give you any information about the number of tokens in the other boxes. If you are bidder 1, you know that bidder 2 can only observe the content of the overscored box and cannot observe the content of the underscored box and the boxes that are neither underscored nor overscored. The analogous reasoning applies to bidder 2.

After observing the content of the underscored box, if you are bidder 1, or the content of the overscored box, if you are bidder 2, you submit a bid for the entire item, that is, for the total number of tokens in all 6 boxes. (We will explain in a minute how bids are transformed into payoffs). You do not get to see the bid of the other participant. Instead, you move to round 2.

Round 2

[SCREEN 3]

In round 2, you keep the same role and bid against the same participant as in round 1. The screens that bidders 1 and 2 see are similar to the upper and lower part of the slide. Bidder 1 now sees the number of tokens inside the first two boxes starting from the left, the two underscored boxes. Bidder 2 sees the number of tokens inside the two boxes starting from the right, the two overscored boxes. Notice that the number of tokens in the leftmost and rightmost boxes did not change. This happens because, as we explained before, the tokens inside each box were all drawn at the beginning of the match and do not change between rounds. They are just sequentially revealed to bidders.

After observing the content of the open boxes you submit a bid for the total number of tokens in all 6 boxes. You do not see the bid of the other participant. Instead, you move to round 3.

This process of seeing one more box continues round after round until bidder 1 has seen the content of 5 of the 6 boxes starting from the left and bidder 2 has seen the content of 5 of the 6 boxes starting from the right.

At the end of the 5th round, when all bids have been made, the computer screen displays the total number of tokens in all boxes. This is the value that both participants were bidding for. Then, the computer randomly selects with equal probability one of the 5 rounds. For the round selected, payoffs are computed as follows. The participant who submitted the highest bid in the selected round wins the total number of tokens in all 6 boxes and pays his bid in that round. This payoff can be positive (if the tokens in the boxes exceed the bid), zero or negative. This amount is added or subtracted to the current stock of tokens. The participant who submits the lowest bid pays nothing and obtains nothing; his payoff is zero. The bids in all the other rounds do not count for the payoffs. If both participants submit exactly the same bid, then the computer randomly chooses the winner with equal probability and computes the payoffs just like before.

Remember that in each round you always bid for the tokens in all the boxes, including those for which the content is hidden. You do not have to write the same bid in all rounds of the match. You can increase or decrease your bid from round to round, if you think this will increase your payoff. There are only two restrictions in the bids. First, it has to be an integer number between 0, the minimum value if all the boxes have 0 tokens, and 300, the maximum value if all the boxes have 50 tokens. Second, it cannot exceed your current stock of tokens displayed at the beginning of the match.

When the match is finished, we proceed to the next match. For the next match, the computer randomly reassigns all participants to a new pair, a new role as bidder 1 or bidder 2, and randomly selects the number of tokens to put inside each box. The new assignments do not depend in any way on the past decisions of any participant including you and are done completely randomly by the computer. The assignments are independent across pairs, across participants and across matches. This second match then follows the same rules and payoffs as the first match. Your final payoff in the experiment is equal to your stock of tokens in the end. Basically, it is equal to your initial stock of tokens plus your accumulated payoffs during the experiment.

This continues for 15 matches, after which the experiment ends.

[SCREENS 4 AND 5] - These slides summarize the rules of the experiment.

We will now begin the Practice session and go through one practice match to familiarize you with the

computer interface and the procedures. During the practice match, please do not hit any keys until you are asked to, and when you enter information, please do exactly as asked. Remember, you are not paid for this practice match. At the end of the practice match you will have to answer some review questions.

[AUTHENTICATE CLIENTS]

Please double click on the icon on your desktop that says Seq. When the computer prompts you for your name, type your First and Last name. Then click SUBMIT and wait for further instructions.

[START GAME]

[SCREEN 6]

You now see the first screen of the experiment on your computer. It should look similar to this screen.

[Point to overhead screen as you explain this]

At the top left of the screen, you see your subject ID. Please record that ID in your record sheet. You have been paired by the computer with one other participant and assigned a role as bidder 1 or bidder 2, which you can see on the top left of the screen. The participant you are paired with has been assigned the opposite role (bidder 2 or bidder 1). The pair assignment and role will remain the same for the entire match. You can also see on the top left of the screen that you are in round 1.

If you are bidder 1, you can see the content of the underscored box. While, if you are bidder 2, you can see the content of the overscored box.

If you look at the middle of your screen, you should see “Choose a bid for Round 1”. Please type the day of your birthday and press enter. For example, if you were born on March 12, write 12. This is only for the practice match, in the actual experiment you can type any integer number between 0 and 300.

Round 1 is over. We now move to round 2.

[SCREEN 7]

You now see the screen of round 2. It should look similar to this screen. [Point to overhead screen as you explain this]

If you are bidder 1, you can see the content of two underscored boxes, while if you are bidder 2 you can see the content of two overscored boxes. The contents of the leftmost and rightmost boxes are the same as in the previous round. This happens because the tokens inside each box were all drawn at the beginning of the match.

Please type 300 minus the day of your birthday in the field in front of “Choose a bid for Round 2” and press enter.

Round 2 is over. We now move to round 3.

[SCREEN 8]

You now see the screen of round 3. It is the same as the previous rounds, except that both you and the other participant can see the content of one more box. Please type the month of your birthday in the field in front of “Choose a bid for Round 3” and press enter.

Round 3 is over. We now move to round 4.

[SCREEN 9]

You now see the screen of round 4. It is the same as the previous rounds, except that both you and the other participant can see the content of one more box. Notice that the two boxes in the middle are both underscored and overscored. This means that both you and the other participant can observe their content. Please type 200 + the month of your birthday in front of “Choose a bid for Round 4”.

Round 4 is over. We now move to round 5.

[SCREEN 10]

You now see the screen of round 5. It is the same as the previous rounds, the only difference is that you can see the content of one more box. Please type the first three digits of the year you were born in the field in front of “Choose a bid for Round 5” and press enter.

Round 5 is over. Now the computer will randomly select one the rounds.

[SCREEN 11]

The selected round is highlighted in yellow. The participant with the highest bid will collect this amount and pay his/her bid. The participant with the lowest bid has a payoff of zero in this match.



The bottom part of your screen contains a table summarizing the results for all matches you have participated in. This is called your history screen. It will be filled out as the experiment proceeds. It only shows the results from your pair, not the results from any of the other pairs. Now click "Continue". The practice match is over. Please complete the quiz. Raise your hand if you have any question.

[WAIT for everyone to finish the Quiz]

Are there any questions before we begin with the paid session? We will now begin with the 15 paid matches. Please pull out your dividers. If there are any problems or questions from this point on, raise your hand and an experimenter will come and assist you.

[START MATCH 1]

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[After MATCH 15 read:]

This was the last match of the experiment. Your payoff is displayed on your screen. Please record this payoff in your record sheet and remember to CLICK OK after you are done.

[CLICK ON WRITE OUTPUT]

Your Total Payoff is this amount plus the show-up fee of \$5. We will pay each of you in private in the next room in the order of your Subject ID number. Remember you are under no obligation to reveal your earnings to the other participants.

Please put the mouse behind the computer and do not use either the mouse or the keyboard. Please remain seated and keep the dividers pulled out until we call you to be paid. Do not converse with the other participants or use your cell phone. Thank you for your cooperation.

Could the person with ID number 0 go to the next room to be paid.

[CALL all the participants in sequence by their ID #]