

Tiered housing allocation with pre-announced rankings: an experimental analysis *

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Abstract

We study in the laboratory a variant of the house allocation with existing tenants problem where subjects are partitioned into tiers with hierarchical privileges, and they know their position in the priority queue before making their decision. We evaluate the performance of the modified versions of three well-known mechanisms: Top Trading Cycle, Gale-Shapley and Random Serial Dictatorship with Squatting Rights. For all three mechanisms, we find low rates of participation (around 40%), high rates of truth-telling conditional on participation (around 90%), high proportions of fair allocations (above 90%) and significant efficiency losses. We also observe differences across mechanisms: Random Serial Dictatorship is ranked highest in efficiency and Top Trading Cycle is ranked lowest in fairness. We then show that position in the queue has a positive and significant impact on participation whereas tier has lit-

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tle effect on behavior. Finally, the individual analysis reveals that the majority of subjects who do not play according to the theory still follow discernible patterns of participation and preference revelation.

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JEL Classification: C78, D61, D78.

1 Introduction

This paper studies a version of the *house allocation with existing tenants* problem. In this problem, a set of indivisible goods must be allocated to a number of agents. Money exchanges are not feasible and some agents may be endowed with some of the goods. Examples include the assignment of offices or classes to faculty members, on-campus housing to students, parking spaces to employees, schedules to crew in the transportation industry and others. Following the existing literature, we refer to the indivisible goods as ‘houses’.

We investigate the class of one-sided matching problems where agents are *partitioned into tiers* with different privileges. Indeed, in many cases individuals belong to groups with privileges that are identical within tiers and different across them. For example, in university departments, offices are sequentially allocated to full, associate, assistant and adjunct professors.¹ Undergraduate housing is sequentially allocated to students in their senior, junior, sophomore and freshman year. Crew members choose their preferred schedules as a function of their seniority. Similar tiered structures are present in other situations such as firms, fraternities or the armed forces. In those environments, individuals within a stratum are on a level playing field, but have rights which supersede the rights of agents in the strata below.

A house allocation mechanism is a systematic procedure that assigns houses to prospective tenants, allotting at most one house to each agent. At the outset, some agents are endowed with a house (existing tenants) and some others are not

¹For example, see the office assignment procedure at Providence College, RI (<http://www.providence.edu/academic-affairs/Faculty-Resources/Documents/FacultyOfficePolicy.pdf>) or at the Department of Educational Psychology, Counseling, and Special Education at Penn State University (<http://www.ed.psu.edu/educ/epcse/forms/epcse-office-assignment-policy>).

(newcomers). Similarly, some houses are occupied by an agent while others are vacant. An allocation mechanism can be evaluated on four desirable properties: (a) *Pareto efficiency* (the houses should be optimally allocated given the preferences of the agents); (b) *fairness* (the assignment should respect the priority order); (c) *individual rationality* (an agent should be no worse-off by participating in the mechanism); and (d) *strategy-proofness* (agents should not benefit from misrepresenting their preferences).

Three leading allocation mechanisms have been proposed in the literature, each with different advantages. The most commonly used mechanism in real life applications, *the Random Serial Dictatorship with squatting rights* (RSD), satisfies properties (a,b,d) but not (c), thereby discouraging participation which can imply substantial losses. To get around this problem, Abdulkadiroğlu and Sönmez (1999) propose the *Top Trading Cycles* (TTC), a mechanism that satisfies properties (a,c,d) but not (b). Under this procedure, an agent may end up with a worse allocation than someone below in the priority queue, a feature that may create tensions between agents due to issues related to envy or fairness. Finally, the well-known *Gale-Shapley* mechanism (GS) of two-sided matching theory (Gale and Shapley, 1962) has a natural counterpart in the housing allocation problem. Guillen and Kesten (2012) show that a mechanism used at one of MIT's undergraduate dormitories (the NH4) is theoretically equivalent to GS in the context of the housing allocation with existing tenants problem. This rule satisfies properties (b,c,d) but not (a). Naturally, Pareto inefficiencies create ex-post incentives for swaps.

With the imposition of a tiered structure, an individual in a higher tier can always expropriate the house of an individual in a lower one. As a result of this new hierarchical structure, two of the four above mentioned properties need to be adjusted. First, fairness must apply only to agents in the same tier. We call it *tiered*

fairness. Second, since property rights across tiers are compromised, individual rationality can no longer be globally guaranteed. Following Title (1998), we consider the weaker notion of *tiered individual rationality*, which incorporates the fact that an agent may be forced to switch to another house if her endowment is preferred by another agent who belongs to a higher tier. The first step of our analysis consists in extending the three mechanisms discussed above to a multi-tiered structure, which we denote tSD, tTTC and tGS respectively.² Under the proper modification of fairness and individual rationality, it is straightforward to show that the multi-tiered mechanisms keep the same theoretical properties as their single tier counterparts: tSD satisfies all but tiered individual rationality, tTTC satisfies all but tiered fairness and tGS satisfies all but Pareto efficiency (Proposition 1). Therefore, in tSD some agents should rationally opt out.³ As for tTTC and tGS, we should observe differences in the final allocations between the two but not in the rates of participation or truthful revelation. Whether these predictions match the empirical behavior of agents in a controlled laboratory environment is the main subject of this research.

Surprisingly, there exist only two main experimental tests on housing markets with existing tenants. The seminal paper by Chen and Sönmez (2002), from now on [CS], compares RSD and TTC in an experiment with 12 agents (9 tenants and 3 newcomers) and 12 houses under asymmetric information.⁴ In that paper, TTC dominates RSD both in terms of efficiency (88% v. 74% of full efficiency) and participation rates (79% v. 47%) whereas no significant differences are found in truth-telling rates conditional on participation (71% v. 74%). More recently, Guillen and Kesten

²Since we inform the subjects of the priority queue before they make any decision, we choose to call it tiered Serial Dictatorship (tSD) instead of tRSD.

³Following the literature, Pareto efficiency is defined from the point of view of the participating agents only. This means that tSD may result in Pareto inefficiencies over the set of all agents.

⁴As a robustness check, in a follow up paper Chen and Sönmez (2004) compare RSD and TTC under complete information (where subjects know the payoffs of everyone). The qualitative results are the same.

(2012), from now on [GK], compares TTC and GS (or, more precisely, the equivalent NH4 version) in an identical setting. Using an ordinal efficiency test, they find that GS is more likely to Pareto dominate TTC than the other way around. GS also yields more participation than TTC (78% v. 48%) and comparable truth-telling rates (80% v. 69%). Neither of these papers discusses the empirical fairness of the mechanisms.

Our paper introduces two main changes in the experimental designs of [CS] and [GK]. First, we consider a hierarchical structure with 12 agents divided into 4 tiers, with 2 tenants and 1 newcomer per tier. As mentioned above, a stratified population is empirically relevant for a large class of applications. It is therefore interesting to determine the impact of such a structure. Second and perhaps most importantly, we communicate to the agents not only their tier but also their positions in the priority queue *before* eliciting their participation decision and ranking of alternatives. From a theory standpoint, this should have no effect on either tTTC or tGS, where participation and truthful revelation are dominant strategies. It should affect participation but not ranking under tSD since agents have extra information about the likelihood of losing their endowed allocation. Interestingly, in real life mechanisms we observe both types of rules. For example, in on-campus housing the ranking is pre-announced at the NH4 dormitory at MIT but not in the Carnegie Mellon dormitories. Similarly, some colleges explicitly require adjunct faculty to observe their position in the priority list when classes are assigned.⁵ Instead, the experimental literature has always assumed no pre-announcement. Whether the announcement has practical effects on choices is an empirical question that our experiment can address.

⁵See for example the class assignment rules at Oakton Community College, IL (<http://www.oaktonafa.org/OCC-AFAcontract2009-13.pdf>) and Santa Barbara Community College (http://ia.sbccc.edu/docs/Archive/1.Adjunct_Procedure_A.pdf).

There are three other methodological changes with respect to the previous papers. First, we compare all three mechanisms (tSD, tTTC, tGS) within the same framework. Indeed, the results in [CS] and [GK] suggest that behavior in the laboratory may vary even under identical protocols. It is therefore desirable to make comparisons between mechanisms in a framework that is as unified as possible. Second, we conduct 6 rounds of the game with an identical payoff matrix and set of players but with random reassignment of preferences over houses and positions in the queue at the end of each round. In practice, these mechanisms are typically played only one or a few times. However, it is important to understand the effect of experience in order to provide policy prescriptions. Repetition also allow us to perform tests of individual behavior. Third, given that fairness is emphasized in the theoretical literature as a desirable property, we study which mechanism dominates the others in that dimension.

Final allocations are similar in tTTC and tGS even though payoffs are constructed in a way that they should be different for two-thirds of the subjects (all the tenants) if they played according to the theory. Allocations differ more in tSD. Performance, on the other hand, is similar under all three mechanisms. In all mechanisms, most allocations are fair (more than 90% of the time), participation rates are low (around 40%) and truthful revelation rates conditional on participation are high (around 90%) resulting in significant losses in efficiency. However, we also find interesting differences relative to the previous literature.⁶ Contrary to [CS], tSD outperforms tTTC in terms of cardinal efficiency and truthful revelation rates. Contrary to [GK], differences between tTTC and tGS are not statistically significant. Overall, tSD— a theoretically inferior mechanism— has the highest efficiency because participation rates under tTTC and tGS are as low as under tSD, and

⁶Because the design is different, results are not directly comparable to [CS] or [GK]. However, we still find it instructive to discuss the behavioral differences.

truthful revelation rates under tTTC and tGS are lower than under tSD. On the other hand and as predicted by theory, tTTC has the greatest proportion of unfair allocations.

The most striking result of our analysis is the highly significant impact of the position in the queue on the decision to opt out. Agents with an unfavorable draw in the priority queue suspect that they are unlikely to improve their endowment and choose to ‘play safe’ and keep their initial allocation. By construction, this is suboptimal under tTTC and tGS. Such conditioning could not be done by subjects if rankings were not pre-announced, so we conjecture that this variation accounts for at least some of the differences with the previous literature, although to reach a firm conclusion one would have to conduct treatments with and without pre-announced rankings. Note that opting out when the position is low could potentially be rationalized if we assumed a fixed cost of opting in, evaluating the alternatives and/or thinking through the game. However, in the paper we do not explore behavioral theories of this sort. In any case, the announcement-dependency of choices has important policy implications that deserve to be explored in further detail. In particular, it raises the issue of which of the two theoretically equivalent designs performs better in real life.

As for the other variants introduced, we do not find any systematic tier effect on the behavior of subjects after controlling for potential gains of participation and opting out payoffs. This may not be surprising to some readers although it did not seem a priori obvious to us. We do not find any change in behavior over the course of the experiment under any mechanism, implying either that inefficiencies are unlikely to be reduced through experience or that subjects need substantially more than six matches to learn. Still, the combination of multiple observations and no learning permits an analysis at the individual level. We find that many subjects never opt in

and few always do, suggesting that almost no subject realizes that participating is a dominant strategy in tTTC and tGS despite the transparency of the instructions. The pattern is the opposite when it comes to preference revelation: more than 80% of subjects always report truthfully and less than 10% misrepresent their preferences more than once, implying that the majority of subjects do realize the benefits (or the simplicity) of truthful revelation. Finally, we obtain an interesting result from the interaction of multiple matches and known priority queues. Indeed, the participation behavior of half of the subjects can be rationalized by a fixed monotone rule of the type “I participate if and only if my position in the queue is x^{th} or above”, with $x \in \{1, 2, 3\}$.

Other related literature.

The theoretical literature on housing allocation includes the ‘Hierarchical Exchange’ mechanism (Pápai, 2000) which characterizes the class of Pareto-efficient, reallocation-proof and group strategy-proof mechanisms. Pycia and Ünver (2009) introduce a new class, the ‘Trading Cycles with Brokers and Owners’, and show that all group incentive compatible and efficient mechanisms belong to that class. Under the tiered environment, Title (1998) develops the ‘Tiered Exchange’ mechanism that characterizes the class of group strategy-proof, tiered individually rational, tiered envy-free and Pareto efficient mechanisms. Kurino (2014) extends the housing allocation problem to a dynamic framework.

In the single tiered environment, the house allocation with existing tenants is also related to other real-life problems like the *school choice problem* (Abdulkadiroğlu, Pathak and Roth, 2005; Abdulkadiroğlu, Pathak, Roth, and Sönmez, 2005; Abdulkadiroğlu and Sönmez, 2003) and the *kidney exchange problem* (Roth, Sönmez and Ünver, 2004 and 2005).⁷ In the school choice problem, students are assigned a

⁷ For a comprehensive survey of the literature see Sönmez and Ünver (2011).

seat at one of the public schools. Students have preferences over schools and schools have exogenously determined priorities over students. In the kidney exchange problem, patients seek a compatible donor. Some patients arrive with a donor (analogous to existing tenants and occupied houses respectively) while some patients do not have a donor (analogous to newcomers). Additionally, some altruistic donors may want to donate a kidney (analogous to vacant houses).

There is also an expanding experimental literature on real-life matching problems. The experimental literature on the school choice problem has received substantial attention: Chen and Sönmez (2006) test the three school choice mechanisms discussed in Abdulkadiroğlu and Sönmez (2003); Pais and Pintér (2008) and Featherstone and Niederle (2011) examine the effect of different information settings; Calsamiglia, Haeringer and Klijn (2010) study the effects of limiting the number of schools that can be ranked; Klijn, Pais and Vorsatz (2013) analyze the role of the intensity of preferences and risk aversion; Ding and Schotter (2013) test the effects of communication between subjects and Braun et al. (2012) study the imposition of quotas in university admissions.

There are some other studies on one-sided matching problems. Olson and Porter (1994) compare four mechanisms in the context of assigning time slots at the Jet Propulsion Laboratory. Krishna and Ünver (2008) study the allocation of students to courses in universities and find that the bid-based systems typically employed are less efficient than the GS mechanism. The only study with a multi-tiered environment is the field experiment by Baccara et al. (2012). A set of vacant offices are offered to faculty members partitioned into three groups. The priority queue of agents within each tier is known to everyone at the outset and the allocation is implemented using the RSD mechanism. In this problem, there are no existing tenants so the concept of tiered individual rationality does not apply. The paper finds that externalities

(institutional, co-authorship and friendship networks) strongly affect the subjects' choices.

Finally, there is also a related literature on experiments in two-sided matching that includes studies on matching professional baseball players to teams (Nalban-tian and Schotter, 1995); new physicians to hospitals (Kagel and Roth, 2000 and Ünver 2005); law clerks to judges (Haruvy, Roth and Ünver, 2006) and the role of information in college admissions (Pais et al., 2011). A survey of the theory can be found in Roth and Sotomayor (1990).

2 Tiered property rights: bumping and compensation

In a tiered environment, members of higher tiers are often granted certain privileges which we express in two ways: (i) subjects in higher tiers get to choose first and (ii) they can expropriate from tenants in lower tiers. While the first property can be incorporated in the standard housing allocation with existing tenants problem by suitably altering the priority queue, the second cannot. This leads us to redefine the notions of fairness and individual rationality. Expropriation is a key distinguishing feature of our model. The examples presented below suggest that this characteristic is present in many real-life situations.

At community colleges across the US, classes (both content and time slot) are allocated sequentially by tiers to full-time faculty, adjunct faculty and probationary adjunct faculty. Expropriation across tiers is typically allowed and is called “bumping”. This can happen either during the initial allocation or, more frequently, when classes are canceled due to low enrollment. If this happens to a full-time faculty member, she may then bump into an adjunct’s class. The bumped adjunct is then reassigned to another class, which may or may not involve further bumping.

There are variations across colleges to the degree of bumping allowed. In the variant that we build into our model, full-time faculty are allowed to bump adjuncts but adjuncts are not allowed to bump other adjuncts. In other words, property rights are guaranteed within a tier but not across tiers.^{8,9}

A similar notion of expropriation exists for some companies in the airline industry where crew members are assigned schedules on the basis of seniority, using algorithms that exploit priorities and allow expropriation. For example, in the interactive Preferential Bidding System,¹⁰ all crew members are assigned an “initial bid” which is typically based on past choices. Members are informed of their seniority and initial bids and can submit new bids (i.e., revise their choice). A senior member can always expropriate from a junior member by bidding on the schedule currently allocated to her junior colleague. When this happens, the junior member loses the schedule and is asked to submit a new bid. Members are encouraged to submit several ordered choices so that bids are automatically adjusted whenever expropriation occurs.

Finally, the issue of expropriation naturally leads to that of compensation of an expropriated tenant. Since a bumped tenant had an endowment to begin with, we want to guarantee her a certain utility. There are many possible compensating mech-

⁸The class allocation rules at Henry Ford Community College, MI capture this aptly: “[...] if I’m a senior adjunct with years and years of service and I want to teach a schedule that was taught in the corresponding semester of the most recent academic year by a senior adjunct with fewer seniority service points, I don’t have a contractual priority over that less senior Senior Adjunct. In other words, once you are a Senior Adjunct, you “own” your schedule in the sense that another Senior Adjunct with more seniority cannot take it from you in normal circumstances. However, a full-timer could take it if one of her classes is canceled.” (see FAQs about Probationary Status, Seniority, and Class Assignment, Henry Ford Community College Adjunct Faculty Organization, full text available at http://www.hfcc-afo.org/afo_documents/afo-hfcc_faq.pdf).

⁹In another variant called “bottom bumping”, a faculty member is allowed to displace the least senior faculty of her own type in the absence of unstaffed courses or when bumping those in the lower tiers is not feasible (see the bumping rights at Suffolk Community College, NY, text available at <http://www.fascc.org/docs/bumping.pdf>).

¹⁰See <http://www.crewingsolutions.com> for the details of the algorithm.

anisms. We decided to use one which is simple to explain and easy to implement: a house randomly chosen from the set of available houses. However and as made evident in the next section, the compensation rule does not affect the theoretical properties of the mechanisms since an agent makes her participation and preference revelation decisions only after receiving her new endowment. Therefore, any other rule, including one that converts her into a newcomer by allocating her a null house should a priori have the same consequences.

3 The model

3.1 Basic definitions

Since we use a multi-tier extension of a model that is standard in the literature, we present it briefly (we refer the reader to Abdulkadiroğlu and Sönmez (1999) for a formal and comprehensive exposition of the single-tier version). A tiered housing allocation problem with existing tenants consists of a finite set of agents, exogenously partitioned into a finite number L of ordered sets or ‘tiers’, indexed by l . In each tier, agents are classified as either existing tenants (already occupying a house) or newcomers (not occupying a house). There is a finite set of houses, some of which are occupied by existing tenants while the others are vacant. We assume that the total number of agents is equal to the total number of houses. Also, there is a list of (strict) preference relations for each agent over all houses. Agents require at most one house and strictly prefer occupying a house rather than not.

As discussed above, we model the privileges associated with belonging to a higher tier in two ways. First, members of higher tiers choose houses before those in lower tiers. Second, a member of a higher tier can expropriate the house occupied by an agent in a lower tier. The outcome of a tiered allocation problem is a ‘matching’, that is, an assignment of houses to agents such that each agent is assigned at most

one house and no house is assigned to more than one agent. The typical properties that a matching mechanism should promote are efficiency, fairness, participation and truthful revelation. However, due to the existence of hierarchical privileges, the standard definitions of fairness and participation need to be adapted to our stratified structure. We describe how these basic properties are defined in a tiered matching mechanism \mathbf{M} .

- (a) A tiered matching is *Pareto efficient* if there is no other matching which assigns each agent a weakly preferred house and at least one agent a strictly preferred house. \mathbf{M} is Pareto efficient if for each problem it selects a Pareto efficient matching.
- (b) A tiered matching is *tiered fair* if, when an agent prefers another agent's assignment, then (i) the other agent belongs to a higher tier, or (ii) the other agent is in the same tier and is ranked higher in the priority queue, or (iii) the other agent is in the same tier and is assigned her own house.¹¹ \mathbf{M} is tiered fair if it always yields a tiered fair matching.
- (c) A tiered matching is *tiered individually rational* if no agent gets a house that is worse than her endowment at the beginning of the allocation process for her tier.¹² \mathbf{M} is tiered individually rational if it always yields a tiered individually rational matching.

¹¹The analogue of this property in single tier is called *justified envy-freeness*. Title (1998) introduces *tiered envy-freeness* to describe a matching where agents prefer their allocation to that of subjects in strictly lower tiers. By definition, all the mechanisms discussed in this paper are *tiered envy-free*, as members of higher tiers receive their allocation before those in lower tiers and can expropriate them.

¹²This corresponds to her initial endowment only if she does not lose it to someone in a higher tier. If she does lose it, then it corresponds to a randomly selected house that was either vacant or previously occupied by someone in a higher tier and vacated. In that sense and following Title (1998), an agent who loses her house to someone in a higher tier may be worse-off with her new allocation, but there is nothing she can do about it.

- (d) Finally, \mathbf{M} is *strategy proof* or incentive compatible in dominant strategies if no agent can benefit by unilaterally misrepresenting her preferences independently of the preferences and announcements of the other agents.

Notice that Pareto efficiency ensures that agents do not want to ex-post exchange their allocations. Tiered fairness corresponds to the (within tier) pairwise stability condition in the college admissions problem. Tiered individual rationality guarantees participation conditional on the endowment inherited *after* the choice of agents in higher tiers. Finally, strategy proofness is defined in dominant strategies, which ensures that truthful revelation should be affected neither by the participation and announcement decision of other players nor by the knowledge of the position in the priority queue.

3.2 The tiered allocation mechanisms

We describe the modified versions of the Top Trading Cycles (tTTC), Gale-Shapley (tGS) and Serial Dictatorship with Squatting Rights (tSD) mechanisms, which incorporate a tier structure and a known position in the priority queue. All three mechanisms have a common structure. The allocation of houses is done sequentially by tiers. We start with tier 1 while participants in tiers 2 to L wait. When the allocation for tier 1 is completed, we proceed to tier 2 while participants in tiers 3 to L wait, and so on.

Starting with the top tier, the allocation for each tier l takes place in the following way. First, all the houses that have been assigned to members of tiers 1 to $l - 1$ are not available anymore. If an existing tenant's house has not been allocated to someone at a higher tier, then the agent continues to occupy her house. If, however, that agent's house has been taken by a member of a higher tier, she is compensated with a randomly selected house that was either vacant or occupied by someone in

a higher tier and vacated. Second, an ordering of agents in tier l , also called a priority queue, is chosen and communicated to all the members of the tier, who also observe the house occupied by each agent in the tier. Third, each existing tenant in the tier decides whether to participate in the allocation mechanism or opt out. Those who opt out are assigned their houses and removed from the process. Fourth, the participating agents (existing tenants who opt in and the newcomers) report their preferences over the available houses. Fifth, using the priority queue and the submitted preferences of the agents, the allocation in tier l takes place according to the mechanism being used. The agents and their allocations are removed from the process, and the same steps are repeated in tier $l + 1$. Notice that setting the priority queue *before* the participation decision and preference revelation of agents makes the comparison between tTTC and tGS sharper by ensuring that the truth-telling equilibrium allocations under these two mechanisms are unique and different from each other.

The procedure always follows these general principles. The next sections describe the modifications included in each of the three mechanisms in order to account for the tiered structure of our problem.

3.2.1 Tiered Top Trading Cycle (tTTC)

Building on Gale’s top trading cycle idea (Shapley and Scarf, 1974), Abdulkadiroğlu and Sönmez (1999) proposed the *top trading cycle* (TTC) mechanism.¹³ Consider the agents in tier l who decide to opt in along with the newcomers. Using the preferences submitted and the exogenous priority queue, the allocation procedure in our tiered version works as follows. Starting from the top of the queue, assign

¹³This version is called “you request my house - I get your turn” in Abdulkadiroğlu and Sönmez (1999) and shown to be theoretically equivalent to Gale’s idea of top trading cycles in the context of housing allocation with existing tenants. This version of TTC has been used in the experiments of [CS] and [GK].

each agent in turn her top choice from among the available houses until someone requests the house of an existing tenant in the same tier. If at that point the existing tenant whose house is demanded is already assigned a house, then do not disturb the procedure. Otherwise, modify the remainder of the ordering by inserting the existing tenant to the top of the queue and proceed. Similarly, insert any existing tenant in tier l who has not yet been assigned a house at the top of the queue once her house is demanded. If at any point, a loop forms, it can only be formed by existing tenants in the same tier where each tenant demands the house of the tenant next in the loop. In such cases remove all agents in the loop by assigning them the houses they demand and proceed. Once all the agents in tier l have received their allocation, move on to tier $l + 1$. When the number of tiers is 1, this procedure reduces to the TTC mechanism proposed by Abdulkadiroğlu and Sönmez (1999, p.251).

3.2.2 Tiered Gale-Shapley (tGS)

[GK] have adapted the GS mechanism to the house allocation with existing tenants problem. We simply extend that mechanism to our tiered environment. For tier l and using the exogenous priority queue, construct a priority ordering for each available house as follows. If the house is vacant or occupied by someone in a lower tier, then the corresponding queue for this house is the same as the priority queue of the agents. If the house is currently occupied by someone in the same tier, then assign the highest priority for this house to the corresponding existing agent, and assign the remaining priorities without changing the relative ordering of the remaining agents. Then, using this priority ordering for each house and the submitted preferences of the agents, apply the following version of the *deferred acceptance algorithm* due originally to Gale and Shapley (1962):

Step 1. Each agent applies to her top choice house. For each house, look at its pool of applicants and tentatively assign the house to the agent with the highest priority according to the priority ordering for that house and reject the rest of the applicants.

In general,

Step k. Each rejected agent applies to her next top choice house. For each house, consider its applicants at this step together with the agent (if any) who is currently tentatively placed to it. Among these, assign the house to the agent with the highest priority according to the priority ordering for that house and reject the rest.

The process for tier l ends when no agent in that tier is rejected. At that point, all tentative assignments are finalized, and we move on to tier $l + 1$.

3.2.3 Tiered Serial Dictatorship with Squatting Rights (tSD)

The serial dictator algorithm is the simplest and most commonly employed one. In each tier l , the tSD determines the allocation by simply working its way down the priority queue, assigning each participating agent, in turn, her top choice from among the remaining available houses. The unassigned houses are then passed on to the next tier.

Under tSD, tenants may not want to participate since they are not guaranteed a house that is at least as good as the one they are assigned at the beginning of the process for their tier, resulting in potential losses. However, since members of a higher tier choose before those in lower tiers, it is tiered fair. Note that once an existing agent decides to participate, truthful preference revelation is a dominant strategy.

3.3 Properties

Allocation mechanisms are evaluated (both theoretically and empirically) according to the properties they satisfy. It is well known that for the case of a single tier, no (one-sided or two-sided) matching mechanism satisfies simultaneously the four properties described in section 3.1 (see e.g. [GK] and the references therein). For the same reason, no mechanism can satisfy the four properties in our tiered environment either. The proposition below describes which properties hold under each of the tiered mechanisms.

Proposition 1 *For any ordering of agents in each tier, the mechanisms satisfy the following properties:*

- *tTTC is Pareto efficient, tiered individually rational and strategy-proof;*
- *tGS is tiered fair, tiered individually rational and strategy-proof;*
- *tSD is Pareto efficient, tiered fair and strategy-proof.*

Proof. The notions of fairness and individual rationality have been adequately modified to be applied to each tier sequentially. The allocation algorithm of each mechanism has also been modified to be applied to each tier sequentially. Since the properties hold for any given tier, the sequential application by tiers implies that they must necessarily hold for the tier modified mechanism. \square

The idea is straightforward. In a stratified population, the notions of fairness and participation must be modified in order to ensure identical privileges within tiers and hierarchical privileges between tiers. The key issue is then to find the proper definition of fairness and participation. Once this is achieved, it is immediate to extend the logic of the single-tier mechanisms to a multi-tiered environment. Not surprisingly, given this procedure, the same properties that have been shown for the single-tier setting also hold in the multi-tier framework. Naturally, whether such

theoretical properties hold in practice is an empirical question. [CS] and [GK] document interesting deviations from the theoretical predictions in the single-tier case. The experimental analysis conducted in the next sections report other deviations in our multi-tier environment with known priorities and repetition.¹⁴

4 Experimental design

Our experiment is designed to compare the tTTC, tGS and tSD mechanisms in terms of participation of existing tenants, truthful preference revelation by participating agents, fairness of the final allocation and aggregate efficiency of the outcome. To this purpose we conduct three treatments which differ exclusively in \mathbf{M} , the allocation mechanism employed.

For each treatment we ran three sessions with twelve subjects per session for a total of 108 subjects. The subjects were undergraduate students at the University of California, Los Angeles who were recruited by email solicitation. All sessions were conducted at the California Social Science Experimental Laboratory (CASSEL). The interaction between subjects was fully computerized using an extension of the open source software package Multistage Games.¹⁵ No subject participated in more than one session.

In each session, participants played a total of 6 matches. In the first match, the twelve participants were randomly assigned a role, labeled 1 to 12, and divided into

¹⁴Since the structure of the decision problem is the same in each tier, *theory* predicts identical behavior across tiers. However, notice that subjects in tier 1 can expropriate other subjects, subjects in tier 4 can be expropriated by other subjects and subjects in tiers 2 and 3 can sometimes expropriate and sometimes be expropriated. This feature of the multi-tiered environment may potentially result in *empirical* differences across tiers.

¹⁵This contrasts with [CS] and [GK] who ran their experiment fully and partly by hand, respectively. We do not think that this minor modification had an impact on the subjects' behavior. On the other hand, having the game fully computerized allowed us to play multiple matches in a relatively short period of time (6 matches in 1 hour).

4 tiers of 3 participants each. Subjects in roles 1-2-3 were in tier 1, subjects in roles 4-5-6 were in tier 2, and so on. Subjects in roles 1-2-4-5-7-8-10-11 were existing tenants while subjects in roles 3-6-9-12 were newcomers. Hence, each tier consisted of two existing tenants and one newcomer. There were twelve different houses to be allocated, labeled A to L.

Table 1 shows the payoff to each participant (in dollars) as a function of the house she holds at the end of the allocation process. The square bracket, $[\cdot]$, indicates that the participant is occupying that particular house at the beginning of the match. Note that due to the existence of hierarchical privileges, existing tenants in tiers 2 to 4 might not be occupying these particular houses when the allocation mechanism reaches their tier, although they will be occupying some house. The payoffs differ from previous experiments and the priority queues are also pre-specified and announced to the participants. Payoffs and queues are carefully chosen with several objectives in mind. First, to facilitate comparisons, we want all the Pareto efficient house allocations to have the same aggregate payoff. In the experiment, there are four Pareto efficient allocations within each tier, giving a total of $4^4 = 256$ Pareto efficient house allocations, all with an aggregate payoff of \$235. Second, the initial payoffs are such that only one tenant (role 10) is occupying her most preferred house. Third, payoffs range from \$4 to \$25 providing significant variation. Fourth and quite importantly, with these payoffs and queues the equilibrium allocations under tTTC and tGS are *unique* and *different* from each other. Again this facilitates comparisons of behavior across mechanisms. Fifth, the least interesting participation decision corresponds to a tenant who is first in the queue. In order to minimize that case, we never put the newcomer last in the queue. As we will develop below, satisfying all these properties has two drawbacks: the payoff from not participating is higher than we would like it to be and the equilibrium outcome under tTTC is

unfair only for some newcomers (about 30% of the population). Unfortunately and despite our best efforts, it was not possible to avoid those shortcomings and still satisfy the five other objectives.

Table 1: Payoff matrix

Tier	Role	Houses											
		A	B	C	D	E	F	G	H	I	J	K	L
1	1	[19]	22	11	5	13	10	7	4	6	17	15	8
	2	25	[22]	5	11	10	13	4	7	8	17	6	15
	3	25	22	13	7	8	15	10	11	5	17	4	6
2	4	5	25	[17]	19	4	6	15	13	10	22	8	11
	5	22	17	25	[19]	6	8	15	5	11	4	10	13
	6	22	17	25	19	6	11	15	4	8	5	13	10
3	7	8	6	11	22	[17]	19	5	10	13	25	15	4
	8	17	4	6	25	22	[19]	11	13	10	8	15	5
	9	17	4	25	10	22	19	5	8	13	6	15	11
4	10	11	5	8	17	10	4	[25]	22	15	13	6	19
	11	13	11	4	10	5	25	8	[17]	15	6	22	19
	12	10	17	5	13	25	4	6	22	15	8	11	19

All treatments are implemented as games of incomplete information. At the beginning of each match, each subject knows her own tier, her position in the priority queue, and her payoff of holding each of the twelve houses. Subjects also know the number of tiers, the number of existing tenants and newcomers in each tier and that the payoff tables of other participants may differ. Finally, when the allocation process reaches their tier, they know which houses are not available anymore, which house is occupied by someone in the same tier with a higher position in the priority queue (if any), which house is occupied by someone in the same tier with a lower position in the priority queue (if any), which houses are occupied by members of

lower tiers, and which houses are vacant. Figure 1 shows a sample screenshot for an existing tenant in tier 2 who has chosen to participate.¹⁶

Figure 1: Sample Screenshot

ID: 8
 TIER: 2
 ROLE: Existing Tenant
 PAYOFF

HOUSE	A	B	C	D	E	F	G	H	I	J	K	L
STATUS	L	NA	V	OS	O	V	NA	NA	L	L	V	L
PAYOFF	18	11	10	17	19	9	12	20	21	13	15	16

You Chose: **IN**
 YOUR POSITION IN THE PRIORITY QUEUE IS: 2
 RANK THE AVAILABLE HOUSES:

HOUSE	A	B	C	D	E	F	G	H	I	J	K	L
RANK		N/A					N/A	N/A				

Submit

In each session, the allocation of houses is done sequentially, by tiers. We start with tier 1 while participants in lower tiers wait. The existing tenants first have the option of keeping their current house (by opting ‘out’) or participating (by opting ‘in’). If they opt out, they keep their own house and the process ends for them. The existing tenants opting ‘in’ and the newcomers are simultaneously asked to submit their preferences over the remaining houses.¹⁷ The participants are assigned houses according to the allocation mechanism \mathbf{M} ($\in \{tTTC, tGS, tSD\}$) employed in that session, and these houses become unavailable for the lower tiers. Once the allocation

¹⁶The status of houses are V (white) for “vacant”, L (orange) for “occupied by someone in a lower tier”, NA (blue) for “not available”, O (green) for “occupied by oneself” and OS for “occupied by someone else in the same tier” (red if in higher position and dark green if in lower position). Subjects held an instructions sheet during the entire experiment to remind them the meaning of the acronyms.

¹⁷When submitting their rankings, the participants know the number of existing tenants and newcomers who have decided to opt in.

for tier 1 is over, we move on to tier 2. If an existing tenant has lost her house to a member of a higher tier, she is compensated with a randomly selected house which was either vacant at the beginning of the match or previously occupied by a member of a higher tier and vacated in the process. The allocation process in tier 2 then follows the same steps as in tier 1. The process continues with tiers 3 and 4, at which point the match ends.

At the end of a match, subjects are randomly reassigned a role (1 to 12). Roles 1-2-3 are always associated with tier 1, 4-5-6 with tier 2, and so on. The payoffs associated with each role are always the same as described in Table 1. Subjects in roles 1-2-4-5-7-8-10-11 are always existing tenants and subjects in roles 3-6-9-12 are always newcomers. The only difference is that the position in the priority queue for a participant in a given role changes from match to match.¹⁸ Once the new roles and priorities are reassigned, subjects play the same game under the same rules and using the same allocation mechanism. In each session, subjects make decisions over a total of 6 matches.

At the beginning of each session, instructions were read by the experimenter standing on a stage in the front of the experiment room. The experimenter fully explained the rules with special emphasis on the details of the allocation procedure and answered all questions. Next, subjects went through a practice match in order to familiarize themselves with the computer interface and procedures. Subjects had to complete an interactive computerized comprehension quiz before they could proceed to the paid matches. Subjects were then asked to make their decisions over 6 matches. After each match, subjects were randomly reassigned a role. At the end of the session, one of the matches was randomly selected and subjects were

¹⁸We changed the ordering of the queue between matches to make sure that even if an individual happens to draw the same role as in a previous match, she still faces a different decision problem. However, the six orderings we selected for the six matches all satisfy the properties described above.

paid privately and in cash their earnings in that match. Each session lasted for an average of 1 hour and the average earnings in each session were \$18 plus a show up fee of \$5. Table 2 summarizes the details of each session. Appendix C provides the instructions used in sessions 1-3 for the tTTC mechanism (instructions for the other sessions are similar and available upon request).

Table 2: Session details

mechanism	session	date	# of subjects	# of matches
tTTC	1	7/20/10	12	6
	2	7/20/10	12	6
	3	7/20/10	12	6
tGS	1	7/21/10	12	6
	2	7/21/10	12	6
	3	7/21/10	12	6
tSD	1	7/22/10	12	6
	2	7/22/10	12	6
	3	7/22/10	12	6

5 Results

We begin with a descriptive analysis of the final allocations and then evaluate the performance of the three mechanisms on four fronts: aggregate efficiency, allocative fairness, participation rates and truthful preference revelation.

5.1 Final allocations

Tables 3, 4 and 5 show the distribution of the final allocation of houses by roles under the three mechanisms as well as the participation rates under each role (IN).

The square bracket, $[\cdot]$, indicates the fraction of final allocations that coincide with initial endowments. For the tTTC and tGS mechanisms, the cells shaded in grey indicate the theoretical prediction.

Table 3: Allocations under tTTC

Tier	Role	Houses												IN
		A	B	C	D	E	F	G	H	I	J	K	L	
1	1	[.94]	.06	0	0	0	0	0	0	0	0	0	0	.22
	2	.06	[.89]	.06	0	0	0	0	0	0	0	0	0	.22
	3	0	.06	0	0	.06	0	0	0	0	.83	.06	0	-
2	4	0	0	[.89]	0	0	0	0	0	0	.11	0	0	.28
	5	0	0	0	[1]	0	0	0	0	0	0	0	0	.39
	6	0	0	.06	0	0	0	.89	0	0	0	.06	0	-
3	7	0	0	0	0	[.83]	.06	0	0	.06	.06	0	0	.50
	8	0	0	0	0	.06	[.94]	0	0	0	0	0	0	.33
	9	0	0	0	0	.06	0	0	.06	.11	0	.72	.06	-
4	10	0	0	0	0	0	0	[.11]	.22	.17	0	0	.50	.72
	11	0	0	0	0	0	0	0	[.72]	0	0	.06	.22	.28
	12	0	0	0	0	0	0	0	0	.67	0	.11	.22	-

This first look at the data highlights four major findings, which we will further investigate later. First, participation rates across mechanisms are low. This is surprising, especially for tTTC and tGS where participation is a dominant strategy. Second, the final allocations under tTTC and tGS are similar despite the fact that predicted outcomes (shaded boxes) are different for all eight tenants. In fact, outcomes are very much driven by the subjects' initial endowments, which are closer to the predictions in tGS than in tTTC.¹⁹ Third, there is little dispersion in the

¹⁹Remember that endowment only coincides with first-best choice for role 10. So participants who play at equilibrium in tGS end up with their initial endowment (except for role 10!) but only after ordering other houses first.

Table 4: Allocations under tGS

Tier	Role	Houses												IN
		A	B	C	D	E	F	G	H	I	J	K	L	
1	1	[.94]	.06	0	0	0	0	0	0	0	0	0	0	.28
	2	0	[.94]	0	0	0	.06	0	0	0	0	0	0	.22
	3	.06	0	0	0	0	.06	0	0	0	.83	0	.06	-
2	4	0	0	[.78]	.06	0	0	0	0	.06	.11	0	0	.39
	5	0	0	.11	[.83]	.06	0	0	0	0	0	0	0	.39
	6	0	0	.11	.11	0	0	.72	0	0	0	.06	0	-
3	7	0	0	0	0	[.94]	0	0	0	.06	0	0	0	.33
	8	0	0	0	0	0	[.89]	0	.06	0	0	.06	0	.39
	9	0	0	0	0	0	0	0	.06	.11	0	.83	0	-
4	10	0	0	0	0	0	0	[.28]	0	.28	0	0	.44	.67
	11	0	0	0	0	0	0	0	[.67]	0	0	0	.33	.50
	12	0	0	0	0	0	0	0	.22	.50	.06	.06	.17	-

choices under tTTC and tGS. For example, we can predict the choice of subjects in tiers 1-2-3 with a 72% to 100% accuracy. Dispersion in choices is much higher for subjects in tier 4 and for all subjects under tSD. Fourth, although participation rates are also low under tSD, the distribution of final allocations is quite different from that of tTTC or tGS. This is due to higher truth-telling rates.

5.2 Efficiency

We compare the efficiency of the three mechanisms using a cardinal measure. To this end, we first define two natural benchmarks: the earnings in any of the Pareto efficient equilibria and the earnings if no existing tenant participates and all the newcomers reveal their preference truthfully (since they are alone in their tier they can trivially pick their preferred house among the remaining ones). Aggregate earnings in these two scenarios are \$235 and \$211 respectively. For comparison, we also

Table 5: Allocations under tSD

Tier	Role	Houses												IN
		A	B	C	D	E	F	G	H	I	J	K	L	
1	1	[.61]	.06	.06	0	0	0	0	0	0	.28	0	0	.56
	2	0	[.83]	0	0	0	0	0	0	0	.17	0	0	.17
	3	.39	.11	.06	0	0	0	0	0	0	0	.44	0	0
2	4	0	0	[.67]	.06	0	0	.17	0	0	.06	.06	0	.22
	5	0	0	0	[.78]	0	0	.22	0	0	0	0	0	.39
	6	0	0	.22	.17	0	0	.56	0	0	0	.06	0	-
3	7	0	0	0	0	[.56]	.17	0	0	.06	.06	.17	0	.61
	8	0	0	0	0	0	[.78]	0	0	0	0	.22	0	.33
	9	0	0	0	0	.44	.06	0	.06	0	0	.44	0	-
4	10	0	0	0	0	0	0	[.06]	.11	.06	0	0	.78	.78
	11	0	0	0	0	0	0	0	[.78]	.17	0	.06	0	.22
	12	0	0	0	0	0	0	0	.06	.72	0	0	.22	-

report the aggregate earnings if subjects play according to theory. These are \$235 under tTTC (by definition, since this mechanism is Pareto efficient) and \$211 under tGS. For tSD, the efficiency if all existing tenants were forced to participate is \$235 (again by definition), and the efficiency is \$211 if all tenants knew the preferences of the other agents and therefore could evaluate the optimality of opting in. With these premises, we can calculate overall empirical efficiency as the ratio of the sum of actual earnings to the earnings in the Pareto efficient equilibria. In order to take into account the effect of endowments, we normalize this ratio by subtracting from both the numerator and the denominator, the sum of earnings in the match if no existing tenant participates and all the newcomers reveal their preference truthfully (in this case \$211). Table 6 reports overall normalized empirical efficiency for each match in each session under each mechanism.

Table 6: Normalized Empirical Efficiency

Session	Match	Efficiency		
		tTTC*	tGS**	tSD***
1	1	-0.08	0.46	0.00
	2	0.00	0.42	0.25
	3	0.25	-0.54	0.42
	4	0.00	0.00	0.79
	5	0.00	0.42	0.46
	6	0.46	0.21	0.00
2	1	0.08	0.00	0.25
	2	0.21	0.71	0.21
	3	0.00	0.00	0.21
	4	0.21	0.00	-0.08
	5	0.21	-0.54	0.25
	6	0.00	0.75	0.25
3	1	0.00	0.00	0.54
	2	-0.29	0.00	0.33
	3	-0.04	0.00	0.00
	4	0.00	0.00	0.58
	5	0.50	0.00	0.46
	6	-0.50	0.00	0.46
Overall		0.06	0.10	0.30

* Theoretical = 1.00; ** Theoretical = 0.00

*** Theoretical if all participate = 1.00, Theoretical under full information = 0.00

Efficiency is low. In fact, in 50% of the matches the empirical efficiency is no greater than the efficiency when no tenant participates. T-tests²⁰ show that the average empirical efficiency is significantly lower than theoretical under tTTC ($p < 0.01$) while there is no statistical difference under tGS ($p = 0.18$). Empirical efficiency under tSD is significantly lower than theoretical conditional on all agents participating ($p < 0.01$) but significantly higher than theoretical under full

²⁰The standard errors are clustered at the session level for all the t-tests reported in this section. Additionally, all the results hold if we do not normalize the efficiency.

information ($p = 0.04$). Comparing the empirical efficiency between the mechanisms, Wilcoxon-Mann-Whitney tests show that while empirical efficiencies are not statistically different between tTTC and tGS ($p = 0.71$), the empirical efficiency of tSD is significantly higher than that of tTTC ($p < 0.01$) and tGS ($p < 0.05$). This comes as no surprise since the analysis of the distribution of final allocations revealed strikingly similar patterns for tTTC and tGS.

A measure of greater interest, however, is that of *conditional* empirical efficiency. The conditional empirical efficiency for a match is calculated as the ratio of the sum of actual earnings to the conditional Pareto efficient earnings. The latter is defined as the sum of the earnings under the Pareto efficient allocation for each tier given the allocations observed in the previous tiers. So, for example, if subjects in tier l deviate from the Pareto efficiency, it affects the Pareto efficient allocation and the corresponding earnings of subjects in tier $l + 1$. Once again, to take into account the effect of endowments, we normalize this ratio by subtracting from both the numerator and the denominator the (conditional) sum of earnings in the match if no existing tenant participates and all the newcomers reveal their preference truthfully. Table 7 reports the normalized conditional empirical efficiency for each match in each session under each mechanism. It also reports the normalized conditional theoretical efficiency for tTTC and tGS, which is calculated as the ratio of the sum of earnings under the allocation subjects would have received had they behaved according to the theory conditional on the observed allocation of the previous tiers in that match to the conditional Pareto efficient earnings and normalized in the same way as before. For tSD, we compute the same two theoretical benchmarks as before but conditional on previous behavior.

Empirical and theoretical efficiency can be compared using a simple t-test. Empirical efficiency is significantly lower than theoretical under tTTC ($p < 0.001$) while

Table 7: Normalized Conditional Efficiency

Session	Match	tTTC		tGS		tSD		
		Th.	Emp.	Th.	Emp.	Th.1	Th.2	Emp.
1	1	1.0	0.13	0.21	0.13	1.0	0.00	0.00
	2	1.0	0.00	0.50	0.46	1.0	0.00	0.25
	3	1.0	-0.08	0.87	0.22	1.0	0.00	0.42
	4	1.0	0.00	0.00	0.00	1.0	0.00	0.79
	5	1.0	0.00	0.58	0.15	1.0	0.00	0.46
	6	1.0	0.46	0.00	0.21	1.0	0.00	0.00
2	1	1.0	0.09	0.00	0.00	1.0	0.00	0.25
	2	1.0	0.21	0.77	0.50	1.0	0.00	0.21
	3	1.0	0.00	0.00	0.00	1.0	0.00	0.21
	4	1.0	0.21	0.00	0.00	1.0	0.00	-0.37
	5	1.0	0.21	0.46	-0.06	1.0	0.74	0.40
	6	1.0	0.00	0.21	0.33	1.0	0.00	0.06
3	1	1.0	0.00	0.00	0.00	1.0	0.00	0.54
	2	1.0	0.00	0.00	0.00	1.0	0.00	0.33
	3	1.0	0.20	0.00	0.00	1.0	0.00	0.00
	4	1.0	0.00	0.00	0.00	1.0	0.00	0.58
	5	1.0	0.43	0.00	0.00	1.0	0.21	0.13
	6	1.0	-0.11	0.00	0.00	1.0	0.00	0.46
Overall		1.0	0.10	0.20	0.11	1.0	0.05	0.26

there is no statistical difference under tGS ($p = 0.20$). Empirical efficiency under tSD is significantly lower than theoretical conditional on all agents participating ($p < 0.01$) but not significantly different than theoretical under full information ($p = 0.18$). Finally, we can compare the empirical efficiency between the mechanisms. Wilcoxon-Mann-Whitney tests shows that while empirical efficiencies are not statistically different between tTTC and tGS ($p = 0.85$), the empirical efficiency of tSD is significantly different than that of tTTC ($p = 0.02$) and tGS ($p = 0.03$). Overall, the results on efficiency and conditional efficiency are similar and summarized as follows.

Result 1 (Efficiency) *Normalized efficiency and conditional efficiency is low in all three mechanisms: 0.06-0.10 in tTTC, 0.10 in tGS and 0.26-0.30 in tSD. It is higher for tSD than for tTTC and tGS.*

The result highlighting the efficiency dominance of tSD is surprising in light of the previous experimental literature. As we will discuss below, it reflects the higher tendency to reveal truthfully under tSD than under either tTTC or tGS. Finally, in Appendix A we evaluate the relative efficiency of the different allocation mechanisms using the *ordinal efficiency test* (OET) introduced by [GK]. With this criterion, we find that no mechanism is more likely to Pareto dominate the others.²¹

5.3 Fairness

The second criterion to compare mechanisms is allocative fairness. Recall from Proposition 1 that tTTC is not tiered fair but tGS and tSD are. Surprisingly, the previous experiments have not studied the empirical fairness of the different mechanisms despite its importance. Table 8 summarizes the theoretical and empirical fairness for each observation of each subject broken down by mechanism and house endowment (tenant vs. newcomer). Recall that for each mechanism we have 3 sessions with 6 matches each and 12 subjects for a total of 216 individual observations, of which two-thirds are existing tenants and one-third are newcomers. Just like we did for efficiency, we construct the theoretical fairness prediction not from an ex-ante viewpoint but, instead, *conditional* on the allocations chosen by subjects in the tiers above. By construction, under tGS and tSD the equilibrium is always tiered fair, independently of the choices by previous subjects. Under tTTC, choices are predicted to be fair for all the 144 observations of existing tenants but only for

²¹Some of the matching literature has used the recombinant estimator proposed in Mullin and Reiley (2006) to test efficiency. We are unable to use this method as, under our sequential design, we do not have the complete strategy profiles of the participants.

10 out of 72 observations of newcomers.

Table 8: Tiered fairness by mechanism and endowment

	tTTC		tGS		tSD	
	cond. theor.	empirical	cond. theor.	empirical	cond. theor.	empirical
tenant	144/144 (1.0)	139/144* (.97)	144/144 (1.0)	136/144*** (.94)	144/144 (1.0)	137/144*** (.95)
newcomer	10/72 (.14)	60/72*** (.83)	72/72 (1.0)	67/72** (.93)	72/72 (1.0)	69/72 (.96)
total	154/216 (.71)	199/216*** (.92)	216/216 (1.0)	203/216*** (.94)	216/216 (1.0)	206/216*** (.95)

(percentages displayed in parenthesis)

*, **, ***: Significantly different from the theoretical prediction at 90%, 95% and 99% confidence level (sign test)

When the conditional theory prescribes full fairness (tGS, tSD and existing tenants under tTTC), the empirical proportion of fair allocations is very high (.93 or above) though, in all but one case, they are statistically lower than the theoretical prediction. Fairness drops but remains high (.83) when the theory predicts low levels of fairness (newcomers under tTTC). The comparison between mechanisms is perhaps more revealing. T-tests of proportions show that for newcomers the proportion of tiered fair allocations is significantly lower under tTTC than under tGS ($p = .04$, one-sided) or tSD ($p = .01$, one-sided) but not significantly different between tGS and tSD ($p = .47$, two-sided), just like predicted by theory. The same t-test for existing tenants shows no significant differences of fairness across the three mechanisms, again as predicted by theory. Overall, the inferiority of tTTC relative to tGS and tSD in tiered-fairness highlighted in the literature is not as strong in our experiment as the theory predicts but it is still significant. The results on fairness can be summarized as follows.

Result 2 (Fairness) *As predicted by theory, the proportion of unfair allocations is very small under tGS and tSD and significantly larger for newcomers under tTTC.*

5.4 Participation rates

Next we compare the mechanisms on the basis of participation rates. Table 9 reports the participation rates under the different mechanisms broken down by session, match, tier and position in the priority queue. In general, the participation rate of existing tenants is low: 36.8% for tTTC, 39.6% for tGS and 41.0% for tSD. T-test of proportions shows that the overall participation rates are not statistically different between the three mechanisms. Furthermore, differences in participation rates between mechanisms are not statistically significant for any given tier, any match, any position in the queue or for existing tenants who have or have not lost their original house. Notice that no-participation may be optimal for some subjects under tSD because participants are not guaranteed their endowments. Under tGS, most tenants are indifferent not ex-ante but at least *in equilibrium* between participating and not since in both cases they end up with their endowment. This could possibly explain the differences in participation rates between our experiment and [GK] under tGS. However, we find no compelling reason for non-participation under tTTC, where subjects could do substantially better by opting in.

Result 3 (Overall Participation) *Participation rates are low in all three mechanisms: 36.8% for tTTC, 39.6% for tGS and 41.0% for tSD. Differences across mechanisms are not statistically significant.*

Next we look at the determinants of participation rates within mechanisms. First, standard t-tests of proportions performed on the data in Table 9 reveals that tenants in tiers 3 and 4 are more likely to participate than tenants in tiers 1 and 2 under all three mechanisms ($p = 0.025$ for tTTC, $p = 0.061$ for tGS, $p = 0.062$

Table 9: Existing tenants' participation decision

		tTTC	tGS	tSD
Session	1	21/48 (.44)	25/48 (.52)	21/48 (.44)
	2	21/48 (.44)	15/48 (.31)	18/48 (.38)
	3	11/48 (.23)	17/48 (.35)	20/48 (.42)
Tier	1	8/36 (.22)	9/36 (.25)	13/36 (.36)
	2	12/36 (.33)	14/36 (.39)	11/36 (.31)
	3	15/36 (.42)	13/36 (.36)	17/36 (.47)
	4	18/36 (.50)	21/36 (.58)	18/36 (.50)
Match	1	6/24 (.25)	9/24 (.38)	7/24 (.29)
	2	11/24 (.46)	8/24 (.33)	10/24 (.42)
	3	9/24 (.38)	7/24 (.29)	8/24 (.33)
	4	8/24 (.33)	9/24 (.38)	11/24 (.46)
	5	9/24 (.38)	10/24 (.42)	11/24 (.46)
	6	10/24 (.42)	14/24 (.58)	12/24 (.50)
Priority	1	29/45 (.64)	24/45 (.53)	29/45 (.64)
	2	9/27 (.33)	11/27 (.41)	14/27 (.52)
	3	15/72 (.21)	22/72 (.31)	16/72 (.22)
Original House	Lost	16/19 (.84)	13/18 (.72)	14/20 (.70)
	Not Lost	37/125 (.30)	44/126 (.35)	45/124 (.36)
Overall		0.37	0.40	0.41

(percentages displayed in parenthesis)

for tSD). Participation is particularly low in tier 1. The test, however, does not control for other factors, such as the payoff of opting out (which depends on her endowment at the beginning of the allocation process for her tier) or the maximum possible gain. Second and again without controlling for other factors, we find that under all mechanisms an existing tenant is more likely to participate when she is higher in the priority queue. On aggregate, participation rates in priority positions 1, 2 and 3 are 61%, 42% and 24% respectively. Also, t-tests of proportions show that participation rates are significantly higher in position 1 than in 2 or 3 for tTTC

($p = 0.01$ and $p < 0.001$), in position 1 than in 3 for tGS ($p = 0.014$) and in positions 1 and 2 than in 3 for tSD ($p < 0.001$ and $p = 0.004$). The result is in accordance to theory for tSD, where participation is a dominant strategy for the first agent in the queue and becomes probabilistically less interesting as the likelihood of losing the endowment to someone higher in the queue increases. Under tTTC and tGS participation is (weakly) dominant independent of the position. Overall, it suggests that participants are reluctant to opt in if they rationally (for tSD) or irrationally (for tTTC or tGS) believe that participation may result in a loss. Third, we analyze the evolution of participation over the course of the experiment. T-tests of proportions reveal no significant differences in participation rates between match 1 and match 6, between matches 1-2 and matches 5-6 or between matches 1-2-3 and matches 4-5-6, under any mechanism. One could a priori think that agents would learn through repetition that participation is always optimal under tTTC and tGS. This does not seem to be the case in our experiment. A possible reason is that we provide limited feedback to our subjects. Indeed, we only inform them of the final vector of allocations and the subject's own payoff. Our study being the first to introduce repetitions, we think it is interesting and surprising to notice the lack of change in behavior between matches. However, future research should study in more detail the determinants of learning.

We now turn to examine other factors that may be affecting the decision of agents to participate. Table 10 shows the number of agents who occupy a house (# tenants) and their participation rate (IN) as a function of two variables: (i) the payoff of opting out (own house), that is, the value of the house they occupy when deciding whether to participate or not, and (ii) the potential gain of opting in (max. gain IN) defined as the maximum possible net gain from participation.

Table 10: Participation as a function of maximum gain in and own house payoff

	tTTC		tGS		tSD	
	# tenants	IN	# tenants	IN	# tenants	IN
Own house						
25	2	.00	5	.20	1	1.00
22	18	.22	18	.22	20	.20
19	64	.42	58	.38	64	.47
17	51	.31	51	.41	51	.35
15	6	.50	9	.67	7	.86
10	1	1.00	-	-	-	-
8	-	-	-	-	1	.00
6	1	1.00	1	1.00	-	-
5	-	-	1	1.00	-	-
4	1	1.00	1	1.00	-	-
Max. gain IN						
≥ 9	3	1.00	3	1.00	1	.00
7-8	6	.50	9	.56	9	.67
5-6	21	.43	21	.43	16	.44
3-4	64	.39	57	.35	65	.42
0-2	50	.26	54	.37	55	.35

Outside option and potential gains have the expected effect: participation by existing tenants increases as the payoff of keeping their own house decreases and as the maximum attainable gain from participation increases. This suggests that even if subjects in tTTC and tGS do not always participate contrary to their best interest, the comparative statics on participation retain a rational flavor. Indeed, their behavior could be rationalized if we assumed that subjects have a fixed cost of evaluating options and choosing an ordering: they will be more likely to spend this effort as the potential net gain of opting in increases.

In order to investigate in more detail the determinants of participation, we perform two logit regressions where the dependent variable is a discrete choice variable that takes value 1 if the existing tenant opts in and 0 otherwise. The first specification uses the maximum possible payoff gain as an explanatory variable (columns 2-4) whereas the second specification uses the payoff of keeping her own house (columns 5-7). The common set of independent variables include dummies for tiers, position in the queue, matches and whether the existing tenant has lost her original house. Table 11 reports the results.

The first specification confirms to a large extent the previous results on mean comparisons and highlights one of the major results of our analysis: after controlling for maximum possible gain, participation is negatively affected by being last in the priority queue. Maximum gain has a positive effect on participation (although only under tTTC). The second specification is also informative. Position in the priority queue has still a significant effect on participation and this effect appears to be stronger under tGS when the subject is in a lower tier. Payoff in own house has a significant negative impact on participation under tTTC. Finally, under either specification, participation is equally likely in the first three than in the last three matches, there is no systematic effect of being in a lower tier and no impact of losing the original house (including session dummies do not change the results). The determinants of participation are summarized in the following result.

Result 4 (Determinants of Participation) *Participation increases when agents are higher in the priority queue. Participation also increases when the maximum possible gain is high and the payoff of opting out is low. It does not change over time or across tiers.*

One advantage of having subjects play multiple matches is that we can study behavior at the individual level. In particular, it is instructive to determine whether

participation is bimodal (some individuals always participating and others never do) or spread. Table 12 presents the frequency of participation as a function of the number of times a subject was assigned the role of existing tenant. There are more subjects who never participate than subjects who always do (26 vs. 10). The lower average participation rate under tTTC reported in Result 3 may be driven by the large number of subjects who always opt out (13 as opposed to 6 in tGS and 7 in tSD). Such behavior may be due to a conservative strategy by individuals with problems to understand the rules governing the allocation mechanism, although such interpretation is speculative.

Further analysis of the individual participation decisions reveals some interesting patterns. As developed above, priority in the queue is a crucial explanatory variable. In Table 13, we focus on monotone strategies based on queue position. We categorize subjects into those who: (i) always participate; (ii) participate only when first or second in the queue; (iii) participate only when first in the queue; (iv) never participate.²² This classification explains the behavior of 44% to 64% of the subjects depending on the mechanism. As discussed above, these types are all characterized by some level of rationality, since they realize that the likelihood of improving their allocation by opting in is increasing in their position in the queue.

Result 5 (Individual Participation) *Only 9% of subjects always participate in the mechanism whereas 24% never do. We can classify 54% of subjects according to a monotone participation rule based exclusively on their ranking in the priority queue.*

²²Some subjects never play in a certain position. So, for example, an agent who chooses IN when first, OUT when second and never plays in third position would be classified as a type (iii). Note that agents who chooses IN when first, OUT when third and who never play in second position could be classified as either (ii) or (iii). We put them in (ii).

5.5 Truthful preference revelation

Truthful preference revelation is a dominant strategy for agents participating in any of the three mechanisms under consideration. Table 14 presents for each mechanism the proportion of truthful announcements by session, tier, match, position in the queue, and house endowment. Even though participants submit rankings over all available houses, we only consider the *relevant rankings*. Indeed, if n agents choose to participate in a given tier, they can only end up in a house ranked 1 to n , so we restrict attention to preference revelation over that set.²³ The overall proportion of truthful revelation is high: 86.4% under tTTC, 87.6% under tGS and 94.7% under tSD. T-tests of equality of proportions show that truthful revelation is higher under tSD than under tTTC ($p = 0.023$) or tGS ($p = 0.045$) whereas no significant differences are found between tTTC and tGS ($p = 0.777$). Higher truth-telling rates are partly responsible for the differences in final allocations between tSD (Table 5) and tTTC or tGS (Tables 3 and 4) and also for the differences in efficiency (Result 1). When we compare truth-telling rates across mechanisms for subsets of the data, the most significant difference is that newcomers are more likely to tell the truth under tSD than tTTC ($p = 0.071$) while participating existing tenants are more likely to tell the truth under tSD than tGS ($p = 0.041$).

Next, we look at the determinants of truthful preference revelation within mechanisms. Given the high rates of truth-telling, differences in behavior when we partition the data are likely not to be significant or to be significant but small in magnitude. For example, under tGS truth-telling rates are higher in positions 1 and 2 than in 3 ($p = 0.101$ and $p = 0.017$ respectively). The position in the priority queue

²³The case where all rankings are considered is discussed in Appendix B. Naturally, as we enlarge the set of houses ranked, the likelihood of non-truthful revelation (on purpose, due to inattention or just due to a typographical mistake) increases. However, we show that the same qualitative properties hold under the full ranking specification.

has no effect under tTTC or tSD although, in the latter case, it is mainly because agents almost always reveal truthfully. Also, in general we do not find statistically significant differences under any mechanism when we look at truth-telling rates by matches or when we compare the behavior of newcomers and existing tenants. As in section 5.4, we also perform logit regressions where the dependent variable is a discrete choice variable that takes value 1 if the agent reveals his ranking truthfully and 0 otherwise. The regressions are not very informative due to the little variation in the truth-telling rates (results omitted for brevity).

Result 6 (Overall Truth-telling) *Truth-telling is high in all three mechanisms: 86.4% for tTTC, 87.6% for tGS and 94.7% for tSD. Truth-telling is higher under tSD than under tTTC or tGS and it is not affected by position in the queue, tier or match.*

As argued before, multiple matches allow us to perform an analysis at the individual level. Table 15 presents the frequency of truth-telling by subjects as a function of the number of times the subject submitted preferences over houses. Less than 10% misreport their preferences more than once whereas 80% always report truthfully. It suggests that the observed aggregate differences in truth-telling across mechanisms are mostly driven by the behavior of a few subjects. We conjecture that the differences may reflect the greater difficulty for some individuals to understand the subtleties of the mechanisms.

In our final individual analysis, reported in Table 16, we classify heuristically into categories the 40 instances of misrepresentation. We find 5 categories with 5 to 10 observations each. In (i), subjects rank one of the vacant houses first and then their true preference order. In (ii), subjects switch their top two choices. In (iii), subjects list their own house first and the others in no discernible order. In (iv), subjects follow some pattern in the rankings but there is not enough data to put them in

a separate category (for example, a subject second in the priority queue submits the first two rankings truthfully and the third one randomly). In (v), subjects with no discernible pattern are lumped together.²⁴ One could argue that (i), (iii) and possibly (iv) follow some reasonable logic, and (v) corresponds to individuals who did not pay attention or did not understand the game. The most puzzling behavior is (ii), which would vaguely correspond to a subject who follows a ‘psychological win’ (but admittedly very strange) behavioral strategy of the type ‘either I win by getting the first ranked choice or I win by getting my truly preferred house.’

Result 7 (Individual Truth-telling) *9% of subjects misreport their preferences more than once and 80% always report them truthfully. Less than 4% of the observations follow an indiscernible pattern.*

6 Conclusion

In this paper we have studied in the laboratory the tiered housing allocation problem with existing tenants. We have evaluated the performance of three well-known mechanisms –Top Trading Cycles, Gale-Shapley and Random Serial Dictatorship– on four fronts: efficiency, fairness, participation and truthful revelation. Contrary to some of the literature, performance is similar across mechanisms with an efficiency advantage for tSD due to higher truthful revelation rates and a fairness disadvantage for newcomers under tTTC. We have also introduced three novelties in our analysis: tiered structure, multiple matches and known priority queues.

Three results have important policy implications for the future design of matching mechanisms in practical settings. First and foremost, announcing the priority ordering before eliciting choices has a significant effect on behavior, even in mechanisms where participation and truthful revelation are dominant strategies. Second,

²⁴These categories are similar (but not identical) to those described in [CS].

playing the same game a few times is unlikely to improve behavior. It would seem necessary to provide substantial feedback to observe significant changes, although more research is needed to test this hypothesis. Finally, the individual analysis suggests that extremely few subjects realize the dominance of participation. At the same time, the majority of individuals who play out-of-equilibrium still follow some “reasonable” strategies.

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Appendix A: Ordinal Efficiency Test

[GK] suggest an *ordinal efficiency test* (OET) to evaluate the relative efficiency of different allocation mechanisms. To compare the allocations under two mechanism, say tTTC and tGS, the test works as follows. We pick each outcome under tTTC and Pareto compare it to each outcome under tGS. We then count the number of times tTTC dominates tGS and the number of times tGS dominates tTTC, and use a sign rank Wilcoxon test for equality of the matched pairs of dominations (comparisons between tTTC and tSD and between tGS and tSD are done in a similar manner).

[GK] argue that the OET is the most appropriate test since we are concerned with eliciting ordinal (not cardinal) preferences from agents. While we concur with the argument, it is also true that (non-equilibrium) deviations from truthful revelation are likely to be affected by the cardinality of payoffs. Hence, empirical performance should take into account such an effect. More importantly, in [GK] the ordering is announced after the agents have made their decision. The efficiency comparison can then be performed with respect to the 10,000 possible priority orderings. In contrast, we announce the ordering before eliciting preferences, so it only makes sense to consider that priority ordering. This results in a dramatic decrease in the number of observations which reduces the statistical power of the OET test. Indeed, since we have 18 matches under each mechanism, we get a total of $18 \times 18 = 324$ paired comparisons. Performing this test, we are unable to reject the null hypothesis of equality in the number of times that one mechanism dominates the others. Out of 324 comparisons, tTTC dominated tGS 37 times and tGS dominates tTTC 14 times ($p = 0.448$); tGS never dominates tSD and tSD dominates tGS 1 time ($p = 0.317$); finally, tTTC never dominates tSD and tSD never dominates tTTC.

Appendix B: preference revelation (full ranking)

In this section we check the robustness of our preference revelation analysis by considering all the rankings submitted by subjects. Naturally, the overall proportion of truthful preference revelation decreases. It becomes 81% under tTTC, 78% under tGS and 82% under tSD, which for tTTC and tSD are still above those found in the previous literature. Table 17 shows the analogue of Table 14 when all rankings are considered. We find that most of the drop in truth-telling happens in tiers 1 and 2. This is not surprising: in higher tiers subjects have to submit rankings over a larger set of available houses, so they are more liable to make “mistakes” after the first few (relevant) rankings.

Overall, there are 385 observations where individuals submit preferences. In 345 cases, subjects reveal truthfully the relevant rankings (Table 14) and in 310 cases they reveal truthfully the entire ranking (Table 17). Therefore, there are 35 observations where truthful rankings are submitted over the relevant ranking but not over the entire one. In Table 18 we take a closer look at these observations. As

we can see, in almost half of these observations subjects make a mistake either in the last two rankings or in the ranking immediately after the relevant ones.

Table 11: Logit models of participation decisions

	tTTC	tGS	tSD	tTTC	tGS	tSD
Tiers 3&4	0.054 (0.532)	0.687 (0.355)	1.332 (0.822)	-0.296 (0.528)	0.320 (0.508)	1.190 (0.659)
Position 2	-0.908 (0.732)	-0.114 (0.544)	-0.138 (0.388)	-0.852 (0.688)	-0.161 (0.547)	-0.122 (0.400)
Position 3	-2.156* (0.973)	-0.631* (0.248)	-1.131* (0.512)	-2.284* (0.962)	-0.838** (0.259)	-1.152** (0.405)
Position 3*Tiers 3&4	1.259 (0.944)	-0.139 (0.181)	-1.227 (0.859)	1.403 (0.932)	-0.182** (0.035)	-1.233 (0.794)
Last 3 matches	0.056 (0.377)	0.645 (0.450)	0.667 (0.362)	0.048 (0.359)	0.651 (0.439)	0.657 (0.382)
Lost house	1.360 (1.365)	0.555 (0.566)	-0.007 (0.820)	1.643 (1.313)	0.526 (0.836)	0.115 (0.688)
Max. possible gain	0.153* (0.078)	0.159 (0.101)	0.081 (0.095)			
Own house payoff				-0.080* (0.034)	-0.195 (0.107)	-0.050 (0.114)
Constant	-0.453 (0.467)	-1.356** (0.354)	-0.780 (0.746)	1.701** (0.131)	3.007 (2.537)	0.479 (1.981)
N	144	144	144	144	144	144
Pseudo R^2	0.191	0.100	0.166	0.186	0.111	0.164

Standard errors clustered at session level in parenthesis.

*, **: Significant at 95% and 99% confidence level, respectively.

Table 12: Frequency of participation

	Participation Frequency tTTC							Participation Frequency tGS							Participation Frequency tSD						
	0	1	2	3	4	5	6	0	1	2	3	4	5	6	0	1	2	3	4	5	6
Tenant Frequency																					
1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	0	-	-	-	-	-
2	3	2	0	-	-	-	-	0	0	1	-	-	-	-	1	1	1	-	-	-	-
3	3	1	2	1	-	-	-	2	5	2	2	-	-	-	2	3	1	1	-	-	-
4	3	1	2	5	0	-	-	3	4	3	2	2	-	-	0	6	1	3	1	-	-
5	3	1	3	0	2	0	-	0	4	2	1	0	0	-	3	2	1	4	1	1	-
6	1	1	0	1	1	0	0	1	1	0	0	1	0	0	0	2	0	0	0	0	0

Table 13: Classification of individual participation behavior

		tTTC	tGS	tSD
(i)	Always IN	1	6*	4
(ii)	IN if 1st or 2nd	7*	3	6*
(iii)	IN only if 1st	2	1	2
(iv)	Always OUT	13	6	7
	Total	23 (64%)	16 (44%)	19 (53%)

* Includes one individual who did not participate when occupying her most preferred house

Table 14: Proportion of truthful preference revelation (relevant ranking)

		tTTC	tGS	tSD
Session	1	38/45 (.84)	40/49 (.82)	45/45 (1.0)
	2	41/45 (.91)	33/39 (.85)	39/42 (.93)
	3	29/35 (.82)	40/41 (.98)	40/44 (.91)
Tier	1	21/26 (.81)	24/27 (.89)	29/31 (.94)
	2	29/30 (.97)	25/32 (.78)	27/29 (.93)
	3	27/33 (.82)	29/31 (.94)	34/35 (.97)
	4	31/36 (.86)	35/39 (.90)	34/36 (.94)
Match	1	14/18 (.78)	19/21 (.91)	19/19 (1.0)
	2	20/23 (.87)	15/20 (.75)	21/22 (.96)
	3	18/21 (.86)	18/19 (.95)	20/20 (1.0)
	4	17/20 (.85)	19/21 (.91)	22/23 (.96)
	5	20/21 (.95)	19/22 (.86)	19/23 (.83)
	6	19/22 (.86)	23/26 (.89)	23/24 (.96)
Priority	1	50/56 (.89)	45/51 (.88)	55/56 (.98)
	2	46/54 (.85)	52/56 (.93)	54/59 (.92)
	3	12/15 (.80)	16/22 (.73)	15/16 (.94)
Endowment	newcomer	60/72 (.83)	64/72 (.89)	67/72 (.93)
	tenant	48/53 (.91)	49/57 (.86)	57/59 (.97)
Overall		0.86	0.88	0.95

(percentages displayed in parenthesis)

Table 15: Frequency of truth-telling (relevant ranking)

	Truth-telling Frequency tTTC							Truth-telling Frequency tGS							Truth-telling Frequency tSD						
	0	1	2	3	4	5	6	0	1	2	3	4	5	6	0	1	2	3	4	5	6
Submission Frequency																					
1	0	4	-	-	-	-	-	0	1	-	-	-	-	-	1	4	-	-	-	-	-
2	1	2	1	-	-	-	-	0	0	7	-	-	-	-	0	0	2	-	-	-	-
3	0	2	1	5	-	-	-	0	0	2	6	-	-	-	0	0	0	9	-	-	-
4	0	0	0	1	6	-	-	0	1	0	1	8	-	-	0	1	0	1	7	-	-
5	0	0	1	0	1	9	-	0	1	0	0	0	3	-	0	0	0	0	2	5	-
6	0	0	0	1	0	0	0	1	0	0	0	0	0	4	0	0	0	0	0	0	4

Table 16: Classification of Misrepresentations

	tTTC	tGS	tSD	Total
(i) Vacant house	5	2	2	9
(ii) Switch-top-two	4	3	1	8
(iii) Random after own	2	4	2	8
(iv) Unclassified	2	3	0	5
(v) Random	4	4	2	10
Total	17	16	7	40

Table 17: Proportion of truthful preference revelation (full ranking)

		tTTC	tGS	tSD
Session	1	36/45 (.80)	38/49 (.78)	40/45 (.89)
	2	37/45 (.82)	29/39 (.74)	30/42 (.71)
	3	28/35 (.80)	34/41 (.83)	38/44 (.86)
Tier	1	16/26 (.62)	17/27 (.63)	22/31 (.71)
	2	28/30 (.93)	21/32 (.66)	18/29 (.62)
	3	26/33 (.79)	28/31 (.90)	34/35 (.97)
	4	31/36 (.86)	35/39 (.90)	34/36 (.94)
Match	1	14/18 (.78)	17/21 (.81)	17/19 (.90)
	2	19/23 (.83)	14/20 (.70)	17/22 (.77)
	3	17/21 (.81)	16/19 (.84)	17/20 (.85)
	4	16/20 (.80)	18/21 (.86)	21/23 (.91)
	5	18/21 (.86)	17/22 (.77)	17/23 (.74)
	6	17/22 (.77)	19/26 (.73)	19/24 (.79)
Priority	1	46/56 (.82)	40/51 (.78)	44/56 (.79)
	2	44/54 (.82)	46/56 (.82)	51/59 (.86)
	3	11/15 (.73)	15/22 (.68)	13/16 (.81)
Endowment	newcomer	57/72 (.79)	55/72 (.76)	58/72 (.81)
	tenant	44/53 (.83)	46/57 (.81)	50/59 (.85)
Overall		0.81	0.78	0.82

(percentages displayed in parenthesis)

Table 18: Misrepresentations after relevant ranking

	tTTC	tGS	tSD	Total
Mistake in last two ranks only	0	4	2	6
Mistake immediately after relevant ranks	3	3	5	11
Other mistakes	4	5	9	18
Total	7	12	16	35

Appendix C: sample of instructions (tTTC mechanism) NOT FOR PUBLICATION

This is an experiment in the economics of decision making and you will be paid for your participation in cash at the end of the experiment. The entire experiment will take place through computer terminals, and all interaction between participants will take place through the computers. You will remain anonymous to me and to all the other participants during the entire experiment; the only person who will know your identity is the Lab Manager who is responsible for paying you at the end. It is important that you not talk or in any way try to communicate with other participants during the experiment. Remember that you are not being deceived and you will not be deceived: everything I tell you is true.

In this experiment we are going to simulate a house allocation process. The procedure and payment rules will be described in detail below. We will start with a brief instruction period. During the instruction period, you will be given a complete description of the experiment and will be shown how to use the computers. You must take a quiz after the instruction period, so it is important that you listen carefully. If you have any questions during the instruction period, raise your hand and your question will be answered so everyone can hear. If any difficulties arise after the experiment has begun, raise your hand, and an experimenter will come and assist you.

Different participants may earn different amounts. What you earn depends partly on your decisions, partly on the decisions of others, and partly on chance. At the end of the session, you will be paid the sum of what you have earned in the experiment and a show-up fee of \$5. Everyone will be paid in private and you are under no obligation to tell others how much you earned. The experiment consists of 6 matches. The procedure in each match is exactly the same and is as follows:

The Procedure is as follows:

- There are 12 participants divided into 4 tiers. Your participation ID and TIER is mentioned on your screen. [SLIDE #2]
- In each tier there are 3 participants. 2 of them are EXISTING tenants, that is, they currently occupy a house. 1 of them is a NEWCOMER who does not have a house yet. In all there are 8 existing tenants and 4 newcomers. Your ROLE of existing tenant or newcomer is also mentioned on your screen. [SLIDE #2]
- There are 12 houses labeled A-L to allocate. [SLIDE #2] Each house must be allocated to one and only one participant.
- Your payoff for the match, denominated in dollars, depends on the house you hold at the end of the match and it is given in the payoff table like this [SLIDE#2]. For example, if you hold house K at the end of the match, then your payoff is \$17.

- Should you be the current tenant of a house, then this fact is also indicated on your computer screen [SLIDE #2]. Note that different participants might have different payoff tables and these payoffs are privately known.
- In the experiment, the allocation of houses is done sequentially, by tiers. We start with tier 1 while participants in lower tiers wait. When the allocation is done for tier 1 we proceed to tier 2 while participants in lower tiers wait and so on.
- The match ends when the allocation process for all the tiers is over and we move on to the next match. For the next match the computer randomly reassigns the tiers and the roles of existing tenants and newcomers. The new assignments do not depend in any way on the past decisions of any participant including you and are done completely randomly by the computer.
- The second match then follows the same rules as the first match. This continues for 6 matches after which the experiment ends.
- At the end of the experiment the computer randomly selects with equal probability one of the 6 matches and your payoff in the experiment is equal to your payoff in the match selected by the computer.

Classification of Houses:

The houses are classified into the following categories indicated by their STATUS [SLIDE #2]. There are 2 main categories: Not Available and Available.

- NOT AVAILABLE houses are houses which have already been assigned and are no longer available for allocation. These are indicated by the color BLUE and labeled as NA.

The AVAILABLE houses can be further classified into:

- If the house is occupied by you, then it is indicated by the color LIGHT GREEN and labeled (O).
- If it is occupied by someone else in your tier then it is labeled as (OS) and colored RED if the tenant is ranked higher than you in the priority queue (we will explain in a minute what this is) and DARK GREEN if the tenant is ranked lower than you in the priority queue.
- If the house is occupied by someone in a tier lower than yours then it is indicated by the color ORANGE and labeled (L).
- If the house is vacant, that is, not occupied by anyone, it is indicated by the color WHITE and labeled (V).

House Allocation is as follows:

Tier 1:

The house allocation process starts with tier 1 while the other tiers wait. Within tier1 the house allocation takes place in the following way.

- All participants in the tier are lined in a pre-determined priority queue. Your position in the queue is indicated on your screen. [SLIDE #2] Note that your position in the queue does not depend on any of your or anyone else's past decisions.
- Existing tenants first, simultaneously, choose between taking part in the allocation process by choosing IN and not taking part by choosing OUT. [SLIDE #2]
 - If you are an existing tenant and choose OUT, you will keep your current house and the allocation process is over for you.
 - If you are an existing tenant and choose IN, you will then need to rank the available houses. [SLIDE #3]
- If you are a newcomer you cannot choose OUT. You need to rank the available houses.
- Note that the participating existing tenants and the newcomers simultaneously submit their rankings and they need to rank all the available houses. No two houses should be given the same rank.
- Once the participating existing tenants and the newcomers have submitted their ranking of the available houses, the house allocation takes place in the following way:
- We proceed from the top of the priority queue. Based on her chosen ranking of the houses, for the first participant in the queue, we look at the status of her top ranked house from among the remaining houses.
- If the Status of the house is L or V,i.e., if the house is occupied by someone in a lower tier or not occupied by anyone, then the participant at the top of the queue is assigned to it. Note that during the allocation process houses occupied by lower tiers are treated in the same manner as those that are vacant.
- Note that an existing tenant vacates her current house, once she is assigned another house.
- If the requested house is not V or L, it means the requested house is the current house of an existing tenant in the tier. In this case, the existing tenant is moved to the top of the priority queue, directly in front of the requester. This way the existing tenant is always guaranteed a house which is at least as good as the house

she is living in, based on her chosen ranking of the houses. The process continues afterwards with the modified queue.

- If a cycle of requests are formed (e.g., I want John's house, John wants your house and you want my house), all members of the cycle are given what they want, and their new houses are removed from the system.
- The process continues until all participants in tier1 are assigned a house.

Tier 2:

- All participants are lined in a pre-determined priority queue. Your position in the queue is indicated on your screen. Note that your position in the queue does not depend on any of your or anyone else's past decisions.
- The houses assigned to members of tier 1 are not available (NA). In tier 2, if an existing tenant's house has not been allocated to someone in tier 1, then the agent continues to occupy her house. If however, the agent's house has been taken by a member of a higher tier, she is compensated with a randomly selected house which was either previously occupied by a member of tier 1 or not occupied by anyone (V).
- Existing tenants first, simultaneously, choose between taking part in the allocation process by choosing IN and not taking part by choosing OUT.
 - If you are an existing tenant and choose OUT, you will keep your current house and the allocation process is over for you.
 - If you are an existing tenant and choose IN, you will then need to rank the available houses.
- If you are a newcomer you cannot choose OUT. You need to rank the available houses.

The rest of the steps are exactly as those for tier 1. When the allocation for TIER 2 is over, we move on to TIER 3. The steps for TIER 3 are exactly the same as those for TIER 2. When the allocation for TIER 3 is over, we move on to TIER 4, where once again, the steps are exactly the same.

EXAMPLE:

We will now go through a simple example to illustrate how the allocation method works. [Slide #4] Suppose that there are six participants in two tiers. Participants 1, 2 and 3 belong to tier 1 while 4, 5 and 6 belong to tier 2. In tier 1, participants 1 and 2 are existing tenants occupying houses W and X respectively, while 3 is a newcomer. In tier 2,

participants 4 and 5 are existing tenants occupying houses Y and Z respectively, while 6 is a newcomer. In addition, houses U and V are vacant.

We start with tier 1. Suppose the pre-determined priority queue is: 3-1-2. Suppose player 1 chooses OUT and player 2 chooses IN. Player 1 is allocated her current house W and the allocation process is over for her. Then suppose that players 2 who chose IN and player 3 who is automatically in because she is a newcomer, given their payoffs, enter the following ranking of houses: [SLIDE #5]

	Participant 2	Participant 3
Rank 1	V	Z
Rank 2	Y	U
Rank 3	U	Y
Rank 4	X	V
Rank 5	Z	X

Then the allocation for Tier 1 takes place in the following manner:

	Priority Queue	Remaining Houses	Top choice among remaining houses and its status	Actions taken at the end of the step
Step 1	3 – 2	X, Y, Z, U, V	Z, status-L	3 gets Z
Step 2	2	X, Y, U, V	V, status-V	2 gets V

Step1: We start with participant 3. His top choice is house Z which has status L. Participant 3 is assigned house Z.

Step 2: Only participant 2 remains. His top choice among the remaining houses is V which is vacant. So participant 2 is assigned house V.

[SLIDE #6] Now we move on to tier 2. Suppose the priority queue is 6-5-4. Since player 5 has lost her house to tier 1 she is compensated by a house randomly chosen from the set of houses that were previously occupied by tier 1 or vacant. Suppose she is compensated by house X. Now suppose the players 4, 5 choose IN and then players 4, 5 and 6 submit the following ranking of houses.

	Participant 4	Participant 5	Participant 6
Rank 1	X	Y	X
Rank 2	Y	U	Y
Rank 3	U	X	U

Then the allocation for Tier 2 takes place in the following manner:

	Priority Queue	Remaining Houses	Top choice among remaining houses and its status	Actions taken at the end of the step
Step 1	6 – 5 – 4	<i>X, Y, U</i>	<i>X</i> , occupied by 5	6-5-4 becomes 5-6-4
Step 2	5 – 6 – 4	<i>X, Y, U</i>	<i>Y</i> , occupied by 4	5-6-4 becomes 4-5-6
Step 3	4 – 5 – 6	<i>X, Y, U</i>	<i>X</i> , occupied by 5	4 gets <i>X</i> , 5 gets <i>Y</i>

Step 1: The priority queue is 6-5-4. Participant 6 has ranked house X as his top choice which is currently occupied by participant 5. Participant 5 is moved to the top of the queue.

Step 2: The modified priority queue is 5-6-4. Participant 5 has ranked house Y as his top choice which is currently occupied by participant 4. Participant 4 is moved to the top of the queue.

Step 3: The modified priority queue is 4-5-6. Participant 4 has ranked house X as his top choice which is currently occupied by participant 5. Now a cycle is created where Participant 4 wants the house of participant 5 and participant 5 wants the house of participant 4. So participant 4 is given house X and participant 5 is given house Y.

Step 4: Now only participant 6 is left. He gets house U.

The following slide summarizes the rules of the experiment: [Read summary slides #7 and #8]

*** PRACTICE SESSION ***

We will now begin the Practice session and go through a practice match to familiarize you with the computer interface and the procedures. During the practice match, please do not hit any keys until you are asked to do so, and when you enter information, please do exactly as asked. Remember, you are not paid for this practice match. At the end of the practice match you will have to answer some review questions. Are there any questions before we begin?

[AUTHENTICATE CLIENTS]

You have just received your first match. Notice your Role and tier.

- The existing tenant occupying house A in Tier 1 will see a screen like this [SLIDE #9]. Notice that you are asked to choose between taking part in the allocation process by choosing IN and not taking part by choosing OUT. In this the Existing tenant is occupying house A which is colored LIGHT GREEN. House B is colored DARK GREEN as it is occupied by someone else in the tier but with a lower position in the priority queue. The houses occupied by participants in tiers 2, 3 and 4 are colored Orange while houses C,F,K and L are colored white as they are not occupied by anyone.

- Newcomer in Tier 1 will see a screen like this [SLIDE #10]. Notice that you are asked to wait for your turn while the existing tenants in the tier choose between taking part in the allocation process or not. Notice that houses A and B are colored RED as they are occupied by participants with higher position in the priority queue. The houses occupied by participants in lower tiers are colored Orange those that are not occupied by anyone are colored white.
- Those not in tier 1 will see a screen like this [SLIDE #11]. Notice that you are asked to wait for your turn. If you are an existing tenant this is indicated on your screen and the house you occupy is colored LIGHT GREEN.
- Existing Tenants in Tier 1 please click "OUT". Notice that you have been allocated the house that you were occupying. For example, the Existing tenant who was occupying house A will see a screen like this [SLIDE #12].
- Newcomer in Tier 1 will see a screen like this [SLIDE #13]. Please give rank 1 to house C, rank 2 to house D, rank 3 to house E... and so on till rank 10 to house L.
- The Newcomer is allocated house C.

Now we have moved to tier 2

- The existing tenant occupying house E in Tier 2 will see a screen like this [SLIDE #14]. Notice that now houses A, B and C have become Not Available as they have already been assigned to someone in tier 1. In this the Existing tenant is occupying house E which is colored LIGHT GREEN. House D is colored RED as it is occupied by someone else in the tier but with a higher position in the priority queue. The houses occupied by participants in tiers 3 and 4 are colored Orange while houses not occupied by anyone are colored white.
- Notice that all participants in tiers 3-4 are asked to wait while we finish the allocation of tier 2. [SLIDE #15] You will also see that houses A, B and C have become Not Available as they have already been assigned to someone in a higher tier. If you are an existing tenant this is indicated on your screen and the house you occupy is colored LIGHT GREEN.
- Existing Tenants in Tier 2 please click IN. [SLIDE #16] Now Existing Tenants and Newcomer in Tier 2 please give rank 1 to house D, rank 2 to house E, rank 3 to house F,... and so on till rank 9 to house L.
- The participant first in the priority queue is allocated house D, the participant second in the priority queue is allocate house E and the participant third in the priority queue is allocated house F.

Now we have moved to tier 3

- The existing tenant occupying house G in Tier 3 will see a screen like this [SLIDE #17]. Notice that now houses D, E and F have also become Not Available as they have already been assigned to someone in tier 2. As the Existing tenant is occupying house G it is colored LIGHT GREEN. House H is colored DARK GREEN as it is occupied by someone else in the tier 3 but with a lower position in the priority queue. The houses occupied by participants in tier 4 are colored Orange while houses not occupied by anyone are colored white.
- Notice that all participants in tiers 4 are asked to wait while we finish the allocation of tier 3. You will also see that houses D, E and F have become Not Available as they have already been assigned to someone in a higher tier. If you are an existing tenant this is indicated on your screen and the house you occupy is colored LIGHT GREEN.
- Existing Tenants in Tier 3 please click OUT. Notice that you have been allocated the house that you were occupying. For example, the Existing tenant was occupying house G will see a screen like this [SLIDE #18].
- Newcomer in Tier 3 will see a screen like this [SLIDE #19]. Newcomer in Tier 3 please give rank 1 to house I, rank 2 to house J, rank 3 to house K and rank 4 to house L.
- The Newcomer is allocated house I.

Now we have moved to tier 4

- All the participants in tier 4 will see that houses G,H and I have also become Not Available.
- The existing tenant who was earlier occupying house I has lost his house but has now been compensated by another house, chosen randomly from the set of vacant houses. For example, if he is compensated with house L then he will see a screen like this [SLIDE #20].
- Existing Tenants in Tier 4 please click IN. [SLIDE #21] Now Existing Tenants and Newcomer in Tier 4 please give rank 1 to house J, rank 2 to house K, rank 3 to house L.
- The participant first in the priority queue is allocated house J, the participant second in the priority queue is allocate house K and the participant third in the priority queue is allocated house L.

***** END OF PRACTICE SESSION *****

The practice match is over. Please complete the quiz. It has 6 questions. If there are any problems or questions from this point on, raise your hand and an experimenter will come and assist you.

[START QUIZ]

[WAIT for everyone to finish the Quiz]

Are there any questions before we begin with the paid session? We will now begin with the 6 paid matches. If there are any problems or questions from this point on, raise your hand and an experimenter will come and assist you.

[START MATCH 1]

[After MATCH 6 read:]

This was the last match of the experiment. Your payoff is displayed on your screen. Please record this payoff in your record sheet. [CLICK ON WRITE OUTPUT]

Your Total Payoff is this amount plus the show-up fee of \$5. We will pay each of you in private in the next room in the order of your Subject ID number. Remember you are under no obligation to reveal your earnings to the other participants.