Platform Competition with Complementary Products*

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Abstract

We characterize the pricing structure in a model of platform competition in which two firms offer horizontally differentiated platforms and two sets of complementors offer products that are exclusive to each platform, respectively. We highlight the presence of indirect network effects: platforms and complementors benefit from the quality and number of firms in their group and suffer from the quality and number of firms in the rival's group through their effects on prices and market share. We then determine the incentives of platforms to subsidize the independent complementors in an equilibrium. We further analyze the incentives of each platform to form a strategic alliance with complementors through contractual exclusivity or technological compatibility, or to integrate with the complementors. Finally, we discuss the welfare consequences of these strategies.

Keywords. Platform competition, complementors, two-sided markets, compatibility, integration, exclusivity, and subsidy.

JEL classification. L13, L15.

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1 Introduction

In the early 2000s, Sony and Toshiba released a new generation of technologically differentiated, non-compatible DVD formats: Blu-ray and HD-DVD. This contest was widely viewed as an updated version of the VCR battle delivered between Betamax and VHS in the 1980s. As in the previous war, the market eventually tipped although this time with a victory for Sony. According to some industry experts, a major reason for Blu-ray's success stemmed from the ability of the Sony group to secure the exclusive participation of a large fraction of complementary firms (complementors) such as movie studios (Sony, 20th Century Fox, Walt Disney, and Warner Brothers) and major retail distributors including Wal-Mart Stores. Just as in the VCR fight, firm alliances and network effects were at least as decisive as consumer preferences and technological differentiation between the respective groups. Similar battles have been renewed more recently between streaming platforms both in music and television. The videogame industry provides another interesting example of the importance of complementors in platform competition. When Microsoft, Sony or Nintendo launch a new generation of their videogame consoles, they typically provide also new games designed exclusively for their console. These games exploit the comparative advantage of their platform (portability, visual graphics, exclusive characters, etc.).

These multi-billion dollar Information Technology industries share important similarities with each other and also with the computer and smartphone industries. First, there is competition between horizontally and vertically differentiated, possibly non-compatible, platforms: high definition DVD players (Blu-ray and HD-DVD), movie streaming sites (Netflix, Hulu, Amazon), music streaming sites (Spotify, Pandora, Apple music), videogame consoles (XBox, Playstation and Switch), computer operating systems (Windows, macOS and Linux), and smartphone operating systems (Android and iOS). Second, the platforms themselves provide only a limited utility to consumers. In fact, a platform is mostly a means to enjoy some complementary products (movies, videogames, software, apps), making the availability of complementors an essential component in the consumers' purchase decision. As an immediate implication, platform providers are likely to produce some of these complementary products as well. More importantly, platforms will fiercely compete to attract the most important and successful independent complementors (movie distributors, game developers, software programmers) present in the market. Third, the profit of each platform and complementor depends on the prices of all other market players via their effects on the relative attractiveness of each platform.

In this paper, we propose a theoretical model of platform competition that captures the main ingredients of the markets described above. The basic elements of our model are the following. There are three types of players: (i) two platforms that offer vertically and horizontally differentiated products; (ii) an oligopolistic market of complementors that produces goods for either platform; and (iii) a continuum of consumers with different preferences over platforms and complementary products. Consumers enjoy both the platform and the complementary goods associated with the platform. However, platforms are essential in the sense that complementary goods can only be enjoyed through them. Also, there is a multi-party pricing structure: each platform charges positive or negative per-unit fees (royalties or subsidies) to the complementors in its group, and both platforms and all the complementors set prices simultaneously and non-cooperatively to consumers. Finally, platforms may or may not own some complementors.

Our setting combines some elements of previous analysis from two strands of the industrial organization literature. First, as in the literature on hardware-software complementarity and network externalities, players in our setting enjoy indirect network benefits: A platform becomes more attractive to consumers as the number of its complementors increases (see Economides and Salop (1992), Church and Gandal (1993, 2005), Economides and Katsamakas (2006), Farrell and Klemperer (2008) and the references therein). In our model, however, platform competition results in a richer structure of interactions: the number of complementors in each platform affects pricing and therefore profits of both platforms and all complementors, and it also affects the equilibrium utility of consumers. We also consider the direct interactions between the platform and its complementors through royalty payments, contractual exclusivity or technological compatibility. Second, as in the fast developing literature on two-sided markets, the value of a platform for one side of the market increases with the number of players in the other side of the market that adhere to it (for instance, see Rochet and Tirole (2002, 2003, 2006, 2011), Caillaud and Jullien (2003), Armstrong (2006), Hagiu (2006), Armstrong and Wright (2007), Choi (2010), Weyl (2010), Jullien (2011), White and Weyl (2016), Hagiu and Jullien (2014), Hagiu and Wright (2015), Jullien and Pavan (2019) and Tan and Zhou (2021) for theoretical analyses, and Rysman (2004), Lee (2013), Chao and Derdenger (2013), Derdenger (2014) for empirical studies). Because of this positive externality, optimal pricing by platforms involves cross-subsidization. Our model, however, departs from this literature in some important respects. On the one hand, we introduce oligopolistic competition in one side of the market and direct payments between players at both ends. This feature changes the nature of competition and the determination of the price equilibrium. On the other hand, to focus our analysis, we assume that the fraction of players that adhere to each platform in the complementors' side is fixed and exogenously given (e.g., due to technological considerations). We argue that our setting captures better the market structure of the industries described above and is perhaps less compelling for analyzing other examples of two-sided markets (e.g., internet purchases, dating markets and the market for credit cards). More importantly, our model allows us to study new issues such as the optimal three-way pricing (and its comparative statics), the incentives of platforms to form strategic alliances through technological compatibility or exclusive contracts, and the consequences of these business arrangements for the welfare of consumers.

Our analysis and the main conclusions can be summarized as follows. We first characterize the equilibrium when both platforms and all complementors simultaneously set their prices to consumers. We present the conditions for an interior equilibrium (where platforms share the market) and corner equilibria (where the market is entirely captured by one platform). We show that even in a tipping market, the presence of the second platform affects the price and profits of the firms in the dominant group. We also show that all firms in a group benefit from an increase in the platform quality and the number of complementors that adhere to it. At the same time, they suffer from an increase in the platform quality and number of complementors in the rival group. This is simply because a platform is more desirable to consumers the greater the value (quantity and quality) of the firms in the group relative to the value of the firms in the other group. Higher value here implies the possibility to charge higher prices while keeping consumers' loyalty.

The next step is to introduce the possibility of transfers between platforms and complementors prior to the pricing game between firms and consumers. Because a platform is an essential component of a group, we assume that platforms choose a royalty or subsidy per-unit of output sold by complementors to consumers. From the viewpoint of the complementors, this extra pricing instrument only modifies the marginal cost of production. From the viewpoint of the platforms, it introduces a new trade-off: Charging royalties increases direct revenues; providing subsidies decreases the prices that complementors charge to consumers, thereby making the group more attractive. When the two groups have the same number of complementors, we provide conditions on the demand for complementary products under which subsidies arise in equilibrium. In particular, we find that when the demand is weakly cost-amplifying (i.e., the pass-through function is no less than 1) both platforms provide subsidies to their complementors. The intuition is as follows. Despite that a royalty generates positive revenues per-unit, it leads to excessively high prices for complementary products, thereby reducing too much the demand for these products. On the balance, both platforms have incentives to subsidize their complementors. When the demand is cost-absorbing (i.e., the pass-through function is strictly less than 1), numerical calculations suggest that subsidy is also likely to occur in equilibrium.

We then analyze what happens when a platform owns some or all the complementors in the group. As in two-part tariffs, a platform finds it optimal to price its own complementary goods at marginal cost to avoid quantity distortions and gain from the sale of the platform at higher prices. Because this ownership structure increases the equilibrium market share of the platform, it benefits the independent complementors in the group and hurts all the firms (platform and complementors) in the rival group. We also show that consumers are better-off if platforms own all complementors than if complementors are independent because the decrease in complementors' price offsets the possible increase in platform's price. Finally, we argue that a platform that can make its own complementary goods available to the customers of the rival platform (Word for Apple computers or Microsoft games for Playstation) faces a trade-off between revenues generated directly by extra sales and loss of market share. We show that it is always optimal to put the complementary goods for sale, but at a price that exceeds the monopoly level.

We further study the incentives of firms to reach business agreements. We show that each platform gains from making its technology compatible with complementors in the other group, as it expands the attractiveness of the platform to consumers. Each platform also gains by signing an exclusive agreement with complementors, as it reduces the attractiveness of the rival platform to consumers. Which outcome prevails depends on the profits of complementors under the different arrangements, the relative bargaining power of firms, and the legal and technological constraints on the feasible set of agreements.

The paper is organized as follows. In section 2, we introduce the basic model. In section 3, we derive the consumer demand for the competing platforms and complementors, characterize the price equilibrium, and determine the properties of the equilibrium prices and profits. In section 4, we analyze the effect of introducing direct transfers between platforms and complementors. In section 5, we determine the equilibrium prices and the implications when platforms own some of the complementors. In section 6, we study contractual exclusivity and discuss its welfare implications. In section 7, we briefly discuss some extensions of the model.

2 A model of platform competition

2.1 Players

We consider a model of platform competition with complementary goods or services. In the model, there are three sets of players: platform providers, firms offering complementary goods, and consumers of the platforms and the complementary goods.

Platforms. There are two platform providers, $h \in \{A, B\}$, each offering a platform product which provides some direct utility to consumers as well as an access to complementary products, for which consumers also derive benefits. The two platforms are vertically and horizontally differentiated, as we will specify below. Our focus in this paper is on technological platforms such as high-definition DVD players, videogame consoles, and computer or smartphone operating systems, although our model permits other interpretations.

Complementors. There are a number of firms in the market producing goods that are complementary to the platforms. We call this set of firms as complementors. Complementors can be of two types, $h \in \{A, B\}$. Type-A firms produce complementary goods exclusively for platform A and type-B firms produce complementary goods exclusively for platform B. There are n_A and n_B complementors of types A and B, respectively.

Consumers. Horizontal platform differentiation is modeled using a standard Hotelling setting. There is a continuum of consumers uniformly distributed on the interval $[0, 1]$. We index by $x \in [0, 1]$ the location of consumer x. Horizontal differentiation between platforms is formalized by assuming that A is located at $x = 0$ and B is located at $x = 1$. A consumer (he) incurs a linear transportation cost $t\,z$ when the absolute distance between his location and the location of the platform he buys is z. The parameter $t > 0$ captures the degree of horizontal platform differentiation.

For simplicity of our analysis, we assume that consumers only purchase one platform. Buying platform h provides a fixed, non-negative utility w_h to any consumer. The difference, $\Delta w \equiv w_A - w_B$, captures the degree of vertical differentiation between the two platforms. We will sometimes refer to w_h as the intrinsic quality of platform h. More generally, w_h may be interpreted as a reduced form of consumer benefits to capture the dynamics of group formation and technology adoption. It can thus reflect the potential of the platform to attract more and better complementors in the future.

Perhaps more importantly, a platform provides access to its complementary goods.

A consumer who buys platform A will purchase q_A^i (≥ 0) units of the i^{th} type-A complementary product ($i \in \{1, \ldots, n_A\}$). Similarly, a consumer who buys platform B will purchase q_B^j (≥ 0) units of the j^{th} type-B complementary product ($j \in \{1, \ldots, n_B\}$). In our analysis, it is sometime convenient to refer to platform h and the n_h complementors associated with it as "group h".

2.2 Utility of consumers and profits of platform providers and complementors

Consumers enjoy a utility $u(q)$ from the purchase of q units of a complementary product conditional on the purchase a platform, where $u(q)$ is twice continuously differentiable, $u'(q) > 0$ and $u''(q) < 0$ for all $q \in [0, \bar{q})$, with $u(0) = 0$ and $0 < \bar{q} < +\infty$. Suppose a complementor sets a per-unit price s for its product, with $s \leq u'(0)$. Conditional on the platform, consumers will then demand a quantity $q(s) = \arg\max_{\tilde{q}} [u(\tilde{q}) - s\,\tilde{q}]$, yielding the familiar indirect utility function

$$v(s) = u(q(s)) - sq(s),$$

where, using the envelope theorem, $v'(s) = -q(s) < 0$ and $v''(s) = -q'(s) > 0$.

Given our definition of the above conditional indirect utility from consuming complementary products, we can now present consumer payoffs from consuming both the platform and its complementary products. Denote by P_h the price of platform h and by s_A^i and s_B^j the per-unit prices of the i^{th} type-A and the j^{th} type-B complementary products, respectively. Assume that the utility enjoyed by a consumer is additively separable in the number of complementary goods. The payoff of a consumer located at x who purchases platform A and the associated complementary goods is given by

$$U_A(x) = w_A + \sum_{i=1}^{n_A} v(s_A^i) - P_A - tx \tag{1}$$

Similarly, his payoff of purchasing platform B and associated complementary goods is

$$U_B(x) = w_B + \sum_{j=1}^{n_B} v(s_B^j) - P_B - t(1-x) \tag{2}$$

Note in the above specification that additive separability in utility across platform and complementary products and no budget constraint are assumed to simplify our analysis. It implies that buying more units of one complementary good or a larger number of complementary goods in a group does not affect the consumer's marginal utility of consuming other complementary goods in that group. This also implies that, for a fixed set of prices, the utility of a consumer is necessarily non-decreasing in the number of complementary goods associated with the platform he buys. In this paper, however, we emphasize a second, indirect effect: the utility and therefore the purchase decision of a consumer is affected by the intrinsic quality of each platform as well as the number of complementary goods in each group via their effects on the prices of both platforms, the prices of complementary products within the group, and the prices of complementary products across groups.[1]

Given the behavior of consumers, we can determine the profits of platform providers and complementors. Assume for simplicity that both platform providers have zero marginal costs and all complementors have the same marginal cost c (> 0). Denote by Q_h the fraction of consumers who buy the goods from group h, and assume for the time being no direct pricing between platforms and complementors. The profits of platform providers A and B are

$$\Pi_A = P_A \, Q_A \quad \text{and} \quad \Pi_B = P_B \, Q_B, \tag{3}$$

respectively. Similarly, the profits of the i^{th} type-A and the j^{th} type-B complementors are

$$V_A^i = \pi(s_A^i) \, Q_A \quad \text{and} \quad V_B^j = \pi(s_B^j) \, Q_B, \tag{4}$$

respectively, where $\pi(s) \equiv (s - c)q(s)$ is the profit function of each complementor from a consumer who chooses a particular platform.

Just like consumers, firms are affected by platform quality and the number of complementors in two ways. First, holding all prices constant, the proportion of consumers buying from one group increases as the platform quality and the number of complementors in that group increases or as the platform quality and the number of complementors in the other group decreases. This demand effect is rather straightforward. It captures the idea that the desirability of a group depends on the goods it offers relative to the goods offered by the other group. Second and as emphasized above, platform quality and number of complementors also affect the prices charged and hence the profits of the firms.

Remark. Instead of variable units of demand and homogenous tastes, the model could be reinterpreted in terms of unitary (or fixed) demand and heterogeneous tastes. Formally,

[1] One could consider a more general consumer's utility function (with, for example, imperfect substitutability of complementors within a group) or a fixed budget to be allocated among a subset of complementors. This would not affect the indirect network effects highlighted in the paper but it would significantly complicate the analysis.

suppose that $q \in \{0, \bar{q}\}$ and that each consumer has a privately known taste for the good, θ, which is drawn from a distribution with c.d.f. $F(\theta)$ and p.d.f. $f(\theta)$. Given a per-unit price s, the consumer purchases $\bar{q}$ units of the good if $\theta > s$ and 0 otherwise. From an ex-ante perspective, his expected utility is:

$$v(s) = \int_s^{\bar{\theta}} (\theta - s)\bar{q}\, f(\theta) d\theta \quad \text{with} \quad v'(s) = -\big(1 - F(s)\big)\bar{q} < 0 \quad \text{and} \quad v''(s) = f(s)\bar{q} > 0.$$

The reader can immediately notice the one-to-one relationship between demand function $q(\cdot)$ and taste distribution $F(\cdot)$. In the analysis below, we will discuss the implications under these two alternative formulations. Naturally, the most realistic formalization should incorporate *both* variable units and heterogeneous tastes, which could be formalized as $\theta u(q) - sq$. This would accommodate for example the case of avid video gamers who acquire more titles than their casual peers, without necessarily purchasing the entire collection.

2.3 The structure of the pricing game

In the paper, we analyze several pricing games. In the first pricing game, both platform providers and their exclusively associated complementors set simultaneously and non-cooperatively their prices to consumers. The analysis of the equilibrium of this game is presented in section 3 and constitutes the benchmark of our study. In section 4, we extend the analysis into a two-stage pricing game to include direct interactions between platform providers and their complementors. In the first stage, the two platform providers simultaneously choose the royalties or subsidies applied to their complementors. In the second stage, all firms simultaneously choose the prices charged to consumers. In section 5, we abstract once again from direct pricing between platforms and complementors. Instead, we study a situation in which a platform provider owns some complementary goods and decides the price at which they are sold to consumers and whether to make them compatible with the rival platform. Finally, in section 6 we determine the impact of contractual exclusivity between each platform provider and its complementors.

2.4 Notations and basic assumptions

Before proceeding, we present some notations and discuss basic assumptions on the primitives of the model. First, we focus on the natural scenario where the maximum surplus of offering a complementary good is positive. Formally, $v(c) > 0$. Moreover, define

$$\mu(s) \equiv -\frac{q(s)}{q'(s)}.$$

Recall that $\pi(s) = (s - c)q(s)$. It follows that:

$$\pi'(s) = -q'(s)[\mu(s) - s + c].$$

Assumption 1 $\mu'(s) < 1$ *for all* s.

Assumption 1 implies that per-customer profit function $\pi(s)$ is single-peaked in s, which ensures that the monopoly price of a complementor, $s^M = \arg\max_s \pi(s)$, is uniquely defined. This assumption is satisfied for the class of demand functions commonly used in the industrial organization literature, including linear, constant elasticity and exponential demand functions. If we consider the alternative interpretation of the model based on heterogenous taste and fixed demand, then $\mu(\theta) = [1 - F(\theta)]/f(\theta)$ is the hazard rate. Assumption 1 then means that the consumer's virtual value function, $J(\theta) \equiv \theta - \frac{1-F(\theta)}{f(\theta)}$, is strictly increasing in θ. Once again, this is a common assumption in the literature on auction theory and mechanism design. For our analysis, it can also be useful to define the following function:

$$\eta(s) \equiv \frac{1}{1 - \mu'(s)}.$$

By the definition of $\mu(s)$, s^M and under Assumption 1, it is straightforward to show that $ds^M/dc = \eta(s^M)$, indicating how much an increase in marginal cost gets passed to consumers under monopoly pricing. Hence, $\eta(s)$ is often referred to as the *pass-through* function. Notice that:

$$\frac{d^2 \log q(s)}{ds^2} \lesseqgtr 0 \;\Leftrightarrow\; \mu'(s) \lesseqgtr 0 \;\Leftrightarrow\; \eta(s) \lesseqgtr 1$$

In words, log-concave demand functions are "cost absorbing" with a pass-through rate smaller than 1 whereas log-convex demand functions are "cost amplifying" with a pass-through rate larger than 1.[2] Now, define:

$$\phi(s) \equiv \frac{q(s)\pi(s)}{\pi'(s)}$$

for $s \in [c, s^M)$, with $\phi(c) = 0$ and $\lim_{s \to s^M} \phi(s) = +\infty$. Assumption 1 implies that

[2]See Rochet and Tirole (2011) for the introduction of this terminology and its implications in the credit-card market. Weyl and Fabinger (2013) discuss the importance of the pass-through function as an economic tool.

$\phi'(s) > 0$ for $s \in [c, s^M)$.[3] As we will see in Section 3.2, this property of $\phi(\cdot)$ ensures that the profit function of each complementor in (4) is single-peaked in its own price variable and hence that the second-order condition for each complementor's profit maximization is satisfied at the interior solution. Function $\phi(\cdot)$ will play a crucial role in determining the equilibrium in the pricing games.

We also impose an assumption on the platform differentiation parameter t. To simplify our presentation, we focus our analysis of platform competition on the case of full market coverage in which all consumers buy one platform or the other. To this end, we define $\bar{t} = \max\{t \in R_+ : 2v(\phi^{-1}(t)) - 3t \geq 0\}$. The monotonicity of $v(\cdot)$ and $\phi(\cdot)$ imply that $\bar{t}$ is strictly positive and uniquely defined.

Assumption 2 $0 < t \leq \bar{t}$.

As we will show in the next section, Assumption 2 implies that the price equilibrium involves full market coverage in the simplest case of $n_A = n_B = 1$ and $w_A = w_B = 0$, which will in turn imply full market coverage when $n_A, n_B \geq 1$ and $w_A, w_B \geq 0$.

3 The pricing game

In this section, we determine the equilibrium of the one-stage pricing game and its properties. We proceed by first deriving the consumer demand for the two platforms and the complementary products.

3.1 Consumers' demand

Denote by $\bar{x}$ the location of the consumer indifferent between buying platform A and buying platform B, i.e.,

$$U_A(\bar{x}) = U_B(\bar{x}) \quad \Leftrightarrow \quad 2t\bar{x} = t + \Delta w - P_A + P_B + \sum_{i=1}^{n_A} v(s_A^i) - \sum_{j=1}^{n_B} v(s_B^j), \tag{5}$$

[3] Formally, for $s \in [c, s^M)$, we have: $\phi(s) = \frac{\mu(s)}{\mu(s)-s+c} \times \pi(s)$. Furthermore

$$\left(\frac{\mu(s)}{\mu(s) - s + c}\right)' = \frac{\mu(s) - (s-c)\mu'(s)}{(\mu(s) - s + c)^2} > \frac{1}{[\mu(s) - s + c]} > 0$$

where the first inequality follows from Assumption 1 and the second inequality is due to single-peakedness of π.

Assuming full market coverage for the moment, given (5) and a uniform distribution of consumers on the Hotelling line, we obtain the demand functions for platforms A and B as $Q_A = \bar{x}$ and $Q_B = 1 - \bar{x}$, respectively. The demand for complementary goods k and l associated with platforms A and B are given by

$$q_A^k = q(s_A^k)Q_A \quad \text{and} \quad q_B^l = q(s_B^l)Q_B. \tag{6}$$

The demand system exhibits some interesting properties. First, goods within each group are complementary and across groups are imperfect substitutes. Second, the demand for each complementary good takes a multiplicative structure, since the good can only be enjoyed by consumers in conjunction with the corresponding platform. This feature also implies that the demand for each platform and for its associated complementary goods are not symmetric. Third, $q_h^k/Q_h = q(s_h^k)$, that is, the ratio of the demand of complementary good k and the platform within the group depends only on the price of that complementary good. Thus, a platform and its complementary good are strong complements.[4] Fourth, $q_h^k/q_h^i = q(s_h^k)/q(s_h^i)$, that is, the ratio of the demand of two complementary goods within a group depends only on the prices of those complementors, a property of Independence of Irrelevant Alternatives. These properties will be helpful in understanding the equilibrium outcome of the game.

3.2 Price equilibrium with full market coverage

We now determine the equilibrium prices. Platforms A and B solve the following maximization problems

$$\max_{P_A} \ P_A Q_A \qquad \text{and} \qquad \max_{P_B} \ P_B Q_B.$$

The profit function of each platform provider is quadratic in its own price and the first-order conditions for interior solutions are given by

$$t + \Delta w + P_B - 2P_A + \sum_{i=1}^{n_A} v(s_A^i) - \sum_{j=1}^{n_B} v(s_B^j) = 0, \tag{7}$$

$$t - \Delta w + P_A - 2P_B - \sum_{i=1}^{n_A} v(s_A^i) + \sum_{j=1}^{n_B} v(s_B^j) = 0, \tag{8}$$

[4] See Mathewson and Winter (1997) for a definition of strong complements and its implications for optimal multi-product monopoly pricing.

respectively. Similarly complementor k in group A and complementor l in group B solve

$$\max_{s_A^k} \ \pi(s_A^k)Q_A \qquad \text{and} \qquad \max_{s_B^l} \ \pi(s_B^l)Q_B.$$

Notice that

$$\begin{aligned}\frac{\partial[\pi(s_A^k)Q_A]}{\partial s_A^k} &= \pi'(s_A^k)Q_A + \pi(s_A^k)\left[-\frac{q(s_A^k)}{2t}\right] \\ &= \pi'(s_A^k)\left[Q_A - \frac{\phi(s_A^k)}{2t}\right].\end{aligned}$$

Assumption 1 implies that $\phi'(s) > 0$ for $s \in (c, s^M)$. Since $v'(s) < 0$, it follows that $Q_A - \phi(s_A^k)/(2t)$ is decreasing in s_A^k. Thus, each complementor's profit function is single-peaked in its own price variable, which ensures that the optimal interior prices for complementors are determined by the following first-order conditions:

$$t + \Delta w + P_B - P_A + \sum_{i=1}^{n_A} v(s_A^i) - \sum_{j=1}^{n_B} v(s_B^j) - \phi(s_A^k) = 0, \tag{9}$$

$$t - \Delta w + P_A - P_B - \sum_{i=1}^{n_A} v(s_A^i) + \sum_{j=1}^{n_B} v(s_B^j) - \phi(s_B^l) = 0. \tag{10}$$

Note from the first-order conditions that the optimal price of each firm is strictly decreasing in the prices of other firms within the group and strictly increasing in the prices of the firms in the other group. In other words, prices within a group are strategic substitutes whereas prices between groups are strategic complements. This feature is a direct implication of the fact that goods within a group are complementary and across groups are imperfect substitutes.

Imposing symmetry on equilibrium prices of complementors within each group yields the following system of equations [**C**]:

$$\begin{aligned}[\mathbf{C}]: \qquad \phi(s_A) &= t + \frac{1}{3}\Big(n_A v(s_A) - n_B v(s_B) + \Delta w\Big), \\ \phi(s_B) &= t - \frac{1}{3}\Big(n_A v(s_A) - n_B v(s_B) + \Delta w\Big), \\ P_A &= \phi(s_A), \\ P_B &= \phi(s_B).\end{aligned}$$

A solution to system **[C]** consists of an interior price equilibrium, which we denote by $(P_A^*, P_B^*, s_A^*, s_B^*)$. From (5), the equilibrium market shares are:

$$Q_A^* = \frac{P_A^*}{2t} \quad \text{and} \quad Q_B^* = \frac{P_B^*}{2t}.$$

Note that an interior equilibrium may not always exist. When the goods in group A generate a significantly higher surplus to consumers than the goods in group B, a corner equilibrium arises, which we denote by $(\tilde{P}_A, \tilde{P}_B, \tilde{s}_A, \tilde{s}_B)$. In this case, there are multiple equilibria, which take the following form:

$$[\tilde{\mathbf{C}}]: \qquad \begin{aligned} \tilde{s}_A &\geq \phi^{-1}(2t), \quad \tilde{P}_A \geq 2t, \\ \tilde{P}_A &= \Delta w + n_A\, v(\tilde{s}_A) - n_B\, v(c) - t, \\ \tilde{s}_B &= c, \\ \tilde{P}_B &= 0. \end{aligned}$$

At each of the corner equilibria, group A captures the entire market of consumers. Firms in group B price at their marginal costs and nevertheless have zero market share ($\tilde{Q}_A = 1$ and $\tilde{Q}_B = 0$).

Assume that $n_A v(c) + w_A \geq n_B v(c) + w_B$. Since $n_h v(c) + w_h$ represents the maximum surplus that group h could generate (without accounting for consumers' transportation costs), the assumption means that group A is generally more desirable than group B: other things being equal, it yields (weakly) higher potential surplus to consumers. Naturally, the assumption is without loss of generality.

We can summarize the equilibrium of the pricing game in the proposition below.

Proposition 1 *(i) (Interior equilibrium) If* $n_A\, v(\phi^{-1}(2t)) + w_A - 3t < n_B\, v(c) + w_B$, *the equilibrium is interior and uniquely determined by* [**C**].

(ii) (Corner equilibria) If $n_A\, v(\phi^{-1}(2t)) + w_A - 3t \geq n_B\, v(c) + w_B$, *there are multiple equilibria in prices satisfying* [$\tilde{\mathbf{C}}$]*, where firms in group A capture the entire market.*

The condition in part (i) of Proposition 1 means that the difference in the quality of platforms and/or in the number of complementors between the two groups is not significant. In this case, there is a unique, interior equilibrium in prices at which both groups capture a positive share of consumers. Notice that the condition for an interior equilibrium is always satisfied in the symmetric case with $n_A = n_B$ and $w_A = w_B$.

When the difference in the quality of platforms and/or in the number of complementors between the two groups is sufficiently large (the condition in part (ii) of Proposition 1), the stronger group has a significant advantage over its rival group. As a result, the prices of the firms in the weaker group are pushed to the levels of their marginal costs and there is no interior equilibrium. There are multiple equilibria in prices by group A, which is determined by group A's market share equal to 1. The lower bound of the price of the complementary goods is determined such that each complementor does not have incentives to increase its price and reduce the market share.

3.3 Properties of the price equilibrium

3.3.1 Properties of the interior equilibrium

We first discuss a number of interesting properties of the interior equilibrium. One of the objectives of our study is to understand what determines the desirability of a group to consumers in the presence of competition. The platform provider can attract consumers by improving the quality of its platform or by engaging in product differentiation to avoid direct competition. An alternative for the platform provider is to increase the number of the complementary goods that are exclusive to the platform. We can formally address these issues by analyzing how the degree of *substitutability between groups* (captured by parameter t) and the degree of *complementarity between groups* (captured by parameters n_h and w_h, given $v(s)$) affect firms' profitability and consumers' welfare.

Suppose that the condition for an interior equilibrium with full market coverage is satisfied, namely, $n_A\, v(\phi^{-1}(2t)) + w_A - 3t < n_B\, v(c) + w_B$. From system $[\mathbf{C}]$, we obtain the following equilibrium profits for firms in group $h \in \{A, B\}$:

$$\Pi_h^* = \frac{[\phi(s_h^*)]^2}{2t} \quad \text{and} \quad V_h^* = \frac{\pi(s_h^*, c)\phi(s_h^*)}{2t}.$$

The following proposition summarizes how the prices, market shares and profits at the interior equilibrium vary with respect to the size and quality of each group.

Proposition 2 *(Properties of the interior equilibrium) As the platform quality or the number of complementors in a group increases, the following holds:*

- *(i) The market share, prices and profits of all firms in that group (platform and complementors) increase, whereas the market share, prices and profits of all firms in the rival group decrease; and* *(ii) The surplus for each consumer increases.*

These comparative statics suggest that firms exert influences both within and across groups. From the consumers' viewpoint, the desirability of a group depends on the quality of its platform and the number of complementary goods associated with it. Therefore, all firms within a group benefit directly from an increase in its size and/or its quality. This benefit translates into higher prices as well as higher market share and profits. At the same time, an increase in size and in quality in a group hurts the firms in the other group who react by setting lower prices but nevertheless lose market share and profits. This is essentially an indirect network effect on the firms' profitability, positive within a group and negative across groups.

All consumers benefit from any increase in the quality of a platform and the numbers of the complementary goods. For consumers who decide to purchase from group A, an increase in size or in quality of group A has a direct effect on their utility since they continue to purchase from that group. An indirect (strategic) effect arises since these consumers have to pay higher prices. From Proposition 2, the indirect effect is offset by the direct effect. For consumers who decide to purchase from the rival group B, an increase in quality or in the number of complementors in group A would make them either switch or continue to purchase from B. If they continue to purchase from B, then they enjoy lower prices of the platform and complementary goods. If they decide to buy from A, it means that they must benefit even more from the switch. To sum up, we show that a market expansion (e.g., the formation of a new independent game company) is beneficial to all consumers, even those who are not directly affected by the group's expansion.

The effect of an increase in the number of complementors in a group on firms' profits is exacerbated when the total number of complementors is given (for example, as in the DVD industry where the number of movie distributors is roughly fixed). Assuming n constant, as the number of complementors in group A increases, the number of complementors in B is reduced. In this case, Proposition 2 implies that the market share, prices and profits of all firms in group A increase while the market share, prices and profits of all firms in group B fall. This also means that complementors have incentives to join the stronger group so that, in the absence of other countervailing forces, the market for complementary goods will have a tendency to tip. The effect on consumers is not as clear-cut. Starting from a symmetric situation ($n_A = n_B$ and $w_A = w_B$), a marginal increase in n_A combined with a marginal reduction in n_B increases the surplus for consumers who bought from group A

and decreases the surplus for consumers who bought from group B. However, the result does not necessarily extend to asymmetric contexts.

As a remark, remember that the utility of consumers is additively separable in the purchase of platform and complementors. This means that the $n_h + 1$ firms in group h are treated in an essentially symmetric way.[5] The difference in the price equations for P_h^* and s_h^* described in system **[C]** stems exclusively from the assumption that the demand for the complementary goods is variable whereas the demand for each platform is unitary.[6] This relation between pricing with unit vs. variable demand may not surprise some readers, although we have not found it made elsewhere in the literature. It holds also in the absence of complementors, suggesting that all the existing results in Hotelling models with unit demand could be easily extended using the $\phi(\cdot)$ function to account for multi-unit purchases.

3.3.2 Properties of corner equilibria

We now discuss properties of the corner equilibria. Suppose that the condition for corner equilibria is satisfied, namely, $n_A\, v(\phi^{-1}(2t)) + w_A - 3t \geq n_B\, v(c) + w_B$. Part (ii) of Proposition 1 implies that there are multiple equilibria. In all equilibria, consumers enjoy the same utility level $w_B + n_B v(c) + t$ (not counting the transportation cost) and all firms in group B make zero profits. While group A captures the entire market, the platform and its complementors have distinct preferences over equilibrium prices. In general there is not a unique way for the platform and complementors in group A to divide the total profits available in the entire market. A reasonable criterion for equilibrium selection is to maximize the total profits of the firms in group A. This leads to the lowest price for the complementors, $\tilde{s}_A = \phi^{-1}(2t)$, and to the highest price for the platform provider, $\tilde{P}_A = \Delta w + n_A\, v(\phi^{-1}(2t)) - n_B\, v(c) - t$. Indeed, given that A has the entire market, it is optimal for the group to lower the price of the complementary goods as much as possible to minimize quantity distortions and use the platform price to extract maximum surplus from consumers. In a sense, the group uses $(\tilde{P}_A, \tilde{s}_A)$ as a two-part tariff. At the selected

[5] As discussed in footnote 1, if we instead assumed imperfect substitutability between complementors or a binding budget constraint, then a consumer would buy the platform and a subset of its complementors, thereby breaking the symmetry between the two types of firms. In other words, the fact that the platform is essential to enjoy the complementors only affects the purchase choices if complementors are linked in the consumer's utility function.

[6] In particular, if we assumed unit demand for complementors ($q(s) = 1$ if $s < \bar{s}$ and $q(s) = 0$ otherwise) and zero marginal cost, then $\phi(s) = s$. This means that, in a symmetric equilibrium, platform and complementors within a group set the same price.

corner equilibrium, platform provider A benefits from an increase in w_A and n_A. By contrast, the complementors in group A (and all the firms in group B) remain unaffected by any change in w_A and n_A.[7]

One implication of the selected corner equilibrium is that even if the firms in the weaker group do not serve any consumer, their presence with marginal cost pricing disciplines the behavior of the firms in the dominant group. To understand better how the presence of a group with zero market share affects consumers and firms in the rival group, we compare the corner equilibrium outcome with the associated equilibrium when there is only one group in the market.[8] Our findings are summarized in the following proposition.

Proposition 3 *(Properties of the corner equilibrium) The presence of group B firms in the market decreases the price and profits of platform A, increases the price and profits of the complementors in group A, and increases consumers' surplus.*

Under condition (ii) in Proposition 1, group A captures the entire market in the presence of group B. As shown in Appendix A3, this implies that group A also captures the entire market in the absence of group B. According to Proposition 3, even though group B firms have zero market share, their presence puts competitive pressure on group A and benefits all consumers. Based on the equilibrium selection criterion discussed above, we find that the presence of group B leads to a reduction in price by platform A but an increase in prices by the complementors in group A. It is intuitive that potential competition affects the prices and profits of operating firms.[9] What appears more surprising and interesting is the differential impact that potential competition has on the strategic choices and profits of firms within a group. In particular, complementors in group A actually benefit from the presence of the rival group. The presence of competition provides another outside option for consumers, which changes the absolute value and sensitivity of the aggregate consumer demand for group A with respect to prices: (a) For any given complementary good price s_A, to maintain the market share equal to 1 in both cases, the platform price is lower in the presence of the rival group than in its absence; (b) The market share in the presence of the rival group is less sensitive to an increase in the price of the

[7] Another possible selection criterion is the set of prices such that group B has a positive market share but arbitrarily close to zero. This is the equilibrium in the boundary when $n_A v(\phi^{-1}(2t)) + \Delta w = n_B v(c) + 3t$.

[8] One possibility is that firms in group B exit the market. To simplify our presentation, in this paper we do not model explicitly the entry and exit decisions of firms.

[9] For example, consider a simple Bertrand competition model with homogeneous goods and different marginal costs. If the monopoly price of the efficient firm is above the marginal cost of the inefficient firm, then the efficient firm sets its monopoly price in the absence of the other firm, and a price slightly below the rival's marginal cost in its presence.

complementary good. These two features imply that, to ensure that each complementor in group A has no incentives to increase its price and reduce the market share from 1, the price of the complementary good in the presence of competition has to be higher than the one without competition.

These results may have practical implications. According to our theory, the movie distributors associated with the Blu-ray format were adversely affected by Toshiba's decision to withdraw the HD-DVD platform from the market (relative to a situation where Toshiba stayed with a small market share) whereas Sony was positively affected by that decision. The evidence on prices appears to point in that direction, at least for the platform. Indeed, immediately after Toshiba's withdrawal, the price of the Blu-ray player increased significantly.[10]

3.4 Endogenous entry of complementors

So far, we have assumed that the number of complementors in each group, n_A and n_B, is exogenously given. However, it may be more realistic to consider endogenous entry in the market for complementors. To address this possibility, we consider a simple model of horizontal differentiation in the complementors' cost of adapting their technology to each platform. Formally, suppose there are N potential complementors located on a Hotelling line and assume that a complementor located at $y \in [0, N]$ suffers costs $\delta\, y$ and $\delta\,(N - y)$ if it adapts its technology to platforms A and B, respectively. In this model, $\delta > 0$ captures the degree of technological differentiation between platforms (it is the analogue for complementors of the parameter t for consumers). Given N sufficiently large (so that, in equilibrium, some complementors always stay out of the market) and a uniform distribution of y, a free-entry equilibrium is then defined by:

$$V_A^*(n_A, n_B) \geq \delta n_A \quad \text{and} \quad V_B^*(n_A, n_B) \geq \delta n_B$$

where $V_h^* \equiv \pi(s_h^*)\phi(s_h^*)/2t$ is the profit for each complementor in group $h \in \{A, B\}$. Assuming symmetry across groups ($w_A = w_B$), we obtain the following result.

Proposition 4 *(i) There is always a symmetric free-entry equilibrium where the number of complementors entering each group is $n_A^* = n_B^* = n^* = \pi(\phi^{-1}(t))/(2\delta)$.*

(ii) If δ is sufficiently small, there is also a tipping equilibrium, where only complementors of one platform enter, and this group captures the entire market.

[10] See for example "Blu-ray player prices hit 2008 highs as competition dwindles" by Mark Rady, *TG Daily*, March 12, 2008.

In a free-entry market with symmetric platforms, complementors with the lowest costs to adapt their technology to a given platform will endogenously enter, creating a symmetric equilibrium with equal market shares of consumers in each group. As the adaptation cost increases, the number of complementors in the market decreases. Also, as the degree of platform differentiation increases, the price and therefore the expected profits of complementors increase, which in turn implies that more complementors choose to enter the market. Formally, $\partial n^*/\partial \delta < 0$ and $\partial n^*/\partial t > 0$. Independently of δ, this equilibrium always exist, as long as N is sufficiently large.

Perhaps more surprisingly, when the technological differentiation parameter δ is small, there is a second class of equilibria characterized by tipping. Indeed, for small δ the increasing cost of adapting the technology to the platform is offset by the increasing returns in the number of complementors in the same group. As a result, one group dominates the market and prevents the complementors in the other group to capture any market share. As in the corner equilibria described in Proposition 3, the presence of the second group constrains the prices and profits that the dominant group obtains. It also limits the number of complementors in the dominant group. More generally, Proposition 4(ii) shows that the endogenous entry of complementors facilitates tipping.

4 Direct interactions between platform and complementors

A more comprehensive and realistic formalization of the game played between platforms, complementors and consumers should incorporate the possibility of transfers between the first two sets of players. In practice, the magnitude and direction of such transfers are unclear. For example, a substantial share of the revenues generated by videogame consoles come from royalties charged to game developers. By contrast, in the computer industry Microsoft is known to subsidize and encourage programmers to develop new software for the Windows platform.

In this section we incorporate an explicit pricing game between platform and complementors. We assume that platform h can impose a (positive or negative) price r_h to each complementor i in its group per unit of output q_h^i sold to consumers. Because this price is likely to result from binding contractual agreements between firms, it seems reasonable to consider a two-stage pricing game. In the first stage, platforms simultaneously choose their prices (r_A, r_B) charged to complementors. In the second stage, after observing (r_A, r_B) all firms simultaneously choose prices (P_A, P_B, s_A^i, s_B^j) charged to consumers. Also, for the remaining of the paper we assume without loss of generality that $w_A = w_B = 0$, so that the asymmetry between groups is entirely captured through n_A and n_B.

For a complementor, the per-unit price r acts as a change in the marginal cost of production from c to $c + r$. With a slight abuse of notation we now denote by $\pi(s, r) = (s - c - r)q(s)$ the per-customer profit of a complementor and let

$$\phi(s, r) \equiv \frac{\pi(s, r)q(s)}{\pi_1(s, r)},$$

so that when $r = 0$, the function $\phi(s, 0)$ is the same as the function $\phi(s)$ defined before.

For a platform, a positive price –from now on referred to as a "royalty"– is a direct mechanism to extract revenues from complementors. A negative price –from now on referred to as a "subsidy"– is an indirect mechanism to extract more revenues from consumers since, other things being equal, it makes the entire group more attractive to them. Overall, as in the two-sided markets literature, when setting the different prices, platforms in our model have to weigh the relative benefits of direct revenues and cross-subsidization. There are also subtle differences with this literature. In particular, the direct pricing between consumers and complementors adds an extra consideration in the platform's pricing decision. Indeed, in the two-sided market literature, modifying the price charged to one side affects the proportion of players that adhere to the platform, and therefore the desirability and willingness to pay for players in the other side (Rochet and Tirole, 2006). Instead, in our setting the fraction of captive players on the complementors side is exogenously fixed (n_A or n_B). The royalty affects the production cost of the complementor and therefore its own price. This, in turn, has an impact on the consumers' decision to buy from one group or the other. Finally, note that platforms have the ability to impose these prices, and therefore affect to some extent the quantities sold by complementors in their group, because they are the essential component of the group. This introduces another asymmetry between platform and complementors.

Consider the case where the market is fully covered. To solve this dynamic game, we first derive the equilibrium outcome in the second stage. Given (r_A, r_B), platforms A and B solve respectively:

$$\max_{P_A} \left(P_A + r_A \sum_{i=1}^{n_A} q(s_A^i) \right) Q_A \qquad \text{and} \qquad \max_{P_B} \left(P_B + r_B \sum_{j=1}^{n_B} q(s_B^j) \right) Q_B,$$

whereas complementor i in group A and complementor j in group B solve:

$$\max_{s_A^i} \ \pi(s_A^i, r_A) Q_A \qquad \text{and} \qquad \max_{s_B^j} \ \pi(s_B^j, r_B) Q_B$$

Imposing symmetry among complementors within each group and solving for the first-order conditions, we obtain the following system of equations $[\mathbf{C}']$ that must be satisfied in an interior price equilibrium:

$$\begin{aligned}
[\mathbf{C}']: \qquad \phi(s_A, r_A) &= t + \frac{1}{3} n_A \rho(s_A, r_A) - \frac{1}{3} n_B \rho(s_B, r_B), \\
\phi(s_B, r_B) &= t - \frac{1}{3} n_A \rho(s_A, r_A) + \frac{1}{3} n_B \rho(s_B, r_B), \\
P_A &= \phi(s_A, r_A) - r_A n_A q(s_A), \\
P_B &= \phi(s_B, r_B) - r_B n_B q(s_B).
\end{aligned}$$

where $\rho(s, r) \equiv v(s) + rq(s)$. Notice that $[\mathbf{C}']$ reduces to $[\mathbf{C}]$ whenever $r_A = r_B = 0$. We denote by $(P_A^{**}, P_B^{**}, s_A^{**}, s_B^{**})$ the solution to system $[\mathbf{C}']$, which are functions of (r_A, r_B). Substituting the equilibrium outcome in the objective functions, the equilibrium market shares and profits for firms in group h are:

$$Q_h^{**} = \frac{\phi(s_h^{**}, r_h)}{2t}, \quad \Pi_h^{**} = \frac{[\phi(s_h^{**}, r_h)]^2}{2t}, \quad V_h^{**} = \frac{\pi(s_h^{**}, r_h)\phi(s_h^{**}, r_h)}{2t} \tag{11}$$

From (11) the equilibrium profit of platform h, Π_h^{**}, is an increasing monotonic transformation of $\phi(s_h, r_h)/(2t)$, or market share. Therefore, the first-stage game between the two platforms is equivalent to:

$$\begin{aligned}
&\max_{r_A} \quad \phi(s_A^{**}, r_A), \\
&\max_{r_B} \quad \phi(s_B^{**}, r_B),
\end{aligned}$$

where s_A^{**} and s_B^{**} are determined by

$$\phi(s_A^{**}, r_A) = t + \frac{1}{3} n_A \, \rho(s_A^{**}, r_A) - \frac{1}{3} n_B \, \rho(s_B^{**}, r_B), \tag{12}$$

$$\phi(s_B^{**}, r_B) = t - \frac{1}{3} n_A \, \rho(s_A^{**}, r_A) + \frac{1}{3} n_B \, \rho(s_B^{**}, r_B). \tag{13}$$

That is, the royalty game in the first stage is equivalent to a constant-sum game in which each platform maximizes its market share. This has an important implication: each platform's market share does not explicitly depend on the rival platform's royalty and the price of the complementary goods of the rival group. When deciding on the optimal royalty rate, a platform only needs to consider a direct effect of its royalty on its market share and an indirect effect through the price of the complementary goods in its own group. Specifically, platform A sets the following derivative equal to 0:

$$\frac{\partial \phi(s_A^{**}, r_A)}{\partial r_A} = \phi_1(s_A^{**}, r_A)\frac{\partial s_A^{**}}{\partial r_A} + \phi_2(s_A^{**}, r_A) = 0. \tag{14}$$

A similar condition applies to platform B.

Applying Cramer's rule to the equilibrium conditions (12) and (13), we can derive $\partial s_A^{**}/\partial r_A$ and $\partial s_B^{**}/\partial r_B$. Let

$$L(s) \equiv \frac{s-c}{s} \quad \text{and} \quad \varepsilon(s) \equiv -\frac{s \cdot q'(s)}{q(s)}$$

be the familiar Lerner's index and the absolute value of the price elasticity of the per-customer demand for a complementary good, respectively. Also, define as before $\eta(s) = \frac{1}{1-\mu'(s)}$ the pass-through function. The equilibrium outcome and its implications can be summarized in the following proposition.

Proposition 5 *(i) An interior subgame perfect equilibrium is determined by (12), (13), and*

$$\eta(s_h^{**}) = \varepsilon(s_h^{**})L(s_h^{**})\left(\frac{s_h^{**} - r_h^{**} - c}{s_h^{**} - c}\right)^2 \tag{15}$$

for each $h \in \{A, B\}$.

(ii) In any interior subgame perfect equilibrium determined by (12), (13), and (15), platform h provides a subsidy to its complementors when the equilibrium price of its complementary goods satisfies

$$\eta(s_h^{**}) > \varepsilon(s_h^{**})L(s_h^{**}) \tag{16}$$

and charges a royalty when the inequality (16) is reversed.

Notice that on the subgame perfect equilibrium path, each platform's royalty rate r_h^{**} is related to its complementor's price s_h^{**} according to (15), and this equilibrium relationship is determined by the demand function $q(s)$ and marginal cost c but does not explicitly depend on n_A, n_B and t.

In a standard single-product monopoly setting, optimal royalty is determined by balancing two effects, a direct effect of increasing revenues and an indirect effect of decreasing the downstream induced demand, which are summarized by the price elasticity of the induced demand. The later is related to the pass-through function of the final consumer demand. When the technology patent holder owns a platform and the rights to license complementary products as in our setting, there is an extra effect of increasing royalty: The price increase of the complementary products reduces the demand for the platform. This effect puts downward pressure on royalty and may lead to subsidy, depending on the relative magnitudes of the above effects. The following corollary illustrates that when the pass-though of the conditional demand for complementary products is large, it is optimal for both platforms to subsidize their complementors.

Corollary 1 *(Subsidy) If $\eta(s) \geq 1$ for all s, then in any interior subgame perfect equilibrium both platforms subsidize their complementors.*

Corollary 1 follows immediately from Proposition 5. Intuitively, when the pass-through of the conditional demand for complementary products is large, an increase in royalty significantly increases the prices and reduces the sales of the complementary products, which would in turn significantly reduce the demand for the platforms, regardless of the degree of competition in our setting. The condition that pass-through is no less than 1 corresponds to weak log-convex (i.e., cost-amplifying) demand functions. It is always satisfied by two well-known classes of demand functions, the constant elasticity demand and the exponential demand.[11] In the model with unitary demand and differential tastes, it corresponds to weakly increasing hazard rate $[1 - F(s)]/f(s)$. Finally, when the pass-through of the conditional demand for complementary products is less than 1, the platforms may still have incentives to subsidize their complementors.[12]

From the first-stage optimization problem notice that the royalty choice is essentially a zero-sum game between platforms. Equilibrium profits are determined by the revenues from royalties and sales to consumers *relative* to the revenues and sales obtained by the rival. Thus, when $n_A = n_B$, any symmetric choice of prices yields the same market

[11]Formally, if $q(s) = s^{-1/\alpha}$ with $0 < \alpha < 1$ then $\eta(s) = 1/(1-\alpha) > 1$. If $q(s) = e^{-\lambda s}$ with $\lambda > 0$, then $\eta(s) = 1$.

[12]We have numerically computed the equilibrium for a large class of demand functions including the linear demand functions which have pass-through rate equal to 1/2, and found that the unique equilibrium involves subsidy in all those cases. Therefore, in our setting competing platforms have strong incentives to subsidize complementors in order to increase the demand for their complementary products and hence enhance the sales of their platforms.

share and profits to both platforms ($Q_h = 1/2$ and $\Pi_h = t/2$). Higher royalties by both platforms implies higher marginal costs for complementors, which result in higher prices of complementors and lower profits. These higher prices also imply smaller consumer surplus. Overall, starting from the Nash equilibrium situation, all parties would (weakly) benefit from a symmetric decrease in the price charged by platforms to complementors. This discussion holds only when the number of complementors is symmetric.

We make a final remark regarding the royalty game. It seems natural to assume the decisions on subsidies/royalties are followed by pricing decisions of platforms and complementors. However, one could argue that platforms may move first and set their prices to consumers at the same time as when they determine the subsidy/royalty rates to complementors. We have considered this possibility in an extension and shown that the basic forces are essentially the same as in Proposition 5, although the analysis is more complex and the exact equilibrium conditions are modified (the technical derivation is omitted for brevity).

5 Ownership of complementary products

In many industries, platforms own a fraction of their complementary products. This ensures that consumers who purchase the platform will enjoy a minimum quantity and quality associated with that group. Such a strategy complements the formation of strategic alliances in the overall objective of tipping the market. This alternative market structure raises several interesting questions. What is the optimal price of the complementors owned by platforms? How does platform ownership affect the profits of the independent complementors? Will a platform sell its complementary products to consumers of the other platform and, if yes, how much will it charge them? The basic model with no direct pricing between platforms and complementors developed in section 3 can be extended to address those issues.

Suppose that platform h owns k_h complementors associated with group h, with $n_h - k_h$ being independent complementors. The optimal prices of the platforms, P_A and P_B, and of the complementors owned by them, $\{s_A^l\}_{l=1}^{k_A}$ and $\{s_B^{l'}\}_{l'=1}^{k_B}$, solve the following problem:

$$\max_{(P_A,\{s_A^l\}_{l=1}^{k_A})} \Pi_A = \left(P_A + \sum_{l=1}^{k_A} \pi(s_A^l) \right) Q_A \text{ and } \max_{(P_B,\{s_B^{l'}\}_{l'=1}^{k_B})} \Pi_B = \left(P_B + \sum_{l'=1}^{k_B} \pi(s_B^{l'}) \right) Q_B \quad (17)$$

Following the same methodology as in section 3 and assuming that all the independent complementors within a group set symmetric prices, we get the following result.

Proposition 6 *(i) Platforms set prices for the complementors they own at marginal cost.*

(ii) As the number of complementors owned by a platform increases, the market share, prices and profits of that platform and of the independent complementors in that group increase, whereas the market share, prices and profits of the platform and of the independent complementors in the other group decrease.

When determining the optimal price for each product, a platform must balance two considerations. Higher prices yield higher revenues per consumer, whereas lower prices attract more consumers and therefore imply more sales of the other products in the group. One difference between platform and complementors is that consumers make a binary choice of the former and variable quantity purchases of the latter. Hence, pricing complementors at marginal cost avoid quantity distortions. All profits are then recouped through the sale of the platform. Notice that the argument is the same as in optimal two-part tariffs, which require unit prices equal to marginal cost and fixed-fees to capture the benefits. This result is also consistent with the finding by Mathewson and Winter (1997) in the context of a monopolist selling multiple products. Here, we show that marginal cost pricing is a dominant strategy for each competing platform selling complementary products. Casual observation suggests that this pricing strategy seems to match the PC industry where operating systems offer many software complementors at discounted prices (or even free) as a way to increase the attractiveness of their platform. A different pattern is observed in the videogame industry, where console producers sell their own games at prices that are roughly equivalent to those of independent game developers, and make a substantial fraction of their profits with those sales. An obvious limitation of our model is the static nature of our framework. A more realistic setting should incorporate the fact that platforms are present for several generations of complementors, thereby creating a commitment problem in the dynamic pricing strategy.

Part (ii) of Proposition 6 is another example of the indirect externalities emphasized all along the paper. By pricing its own complementors at marginal cost, a platform is increasing its desirability for consumers. This translates into a larger market share and therefore higher sales, which benefits the platform but also the independent complementors associated with the group. At the same time, this pricing choice hurts all the firms in the rival group.

Proposition 7 *The welfare of each consumer is higher if platforms own all the complementors than if all the complementors are independent.*

To understand the impact of the platform owning complementary products on consumer welfare, we start with the simple case of symmetry ($n_A = n_B$). In this case, it is straightforward to show that all consumers are better-off under full integration than under no integration. Indeed, they pay lower (marginal cost) prices for complementary products and, since market shares remain at 1/2 in both cases, platform prices are also identical in both cases. The intuition is simple. The integration between the platform and its complementors eliminates the externalities between them (similar to the vertical externality or double marginalization in the traditional upstream-downstream relation), which is beneficial for consumers. Unlike in the standard vertical integration setup, groups cannot recoup any additional profit through platform prices since these are exclusively determined by market shares. The same result applies if both platforms own the same number of complementors $k \in (0, n)$ (that is, consumers' welfare is increasing in k).

The intuition also extends when comparing full and no integration in the asymmetric case, as each platform still sets the prices of complementary products equal to the marginal cost. To illustrate briefly, let $n_A > n_B$, so that A is the most valuable group. In this case, full integration gives group A an advantage in expanding its market share, and therefore also in increasing its platform price at the expense of the market share and platform price of group B. Since group A only captures a fraction of profits with this price increase, consumers of group A are, once again, better-off under full integration. Naturally, consumers of group B are also better-off since all prices are reduced (independently of whether they remain in group B or switch to group A).[13]

Next, we can determine whether a platform has an interest in selling its own complementary products to consumers in the other group, and whether the rival platform should take them in. This may not be technology feasible. Let us assume it is. Then, for the platform selling the complementary products, it implies a trade-off between generating direct revenues and losing market share by making the rival more attractive to consumers. For the platform accepting these new complementary products, the trade-off is between gaining market share and making the rival stronger. Formally, suppose that platform A sells one of its k_A complementary products to consumers in group B. Let z be its price.

[13] The result does not extend directly to asymmetric partial integration. Indeed, s_A increases with k_A and decreases with k_B, so the comparison of the prices for complementary goods between partial integration (k_A, k_B) and no integration $(0, 0)$ depends on the proportion of complementors integrated in each group.

The market share of group A is:

$$Q_A = \left(t + P_B - P_A + \sum_{i=1}^{n_A} v(s_A^i) - \sum_{j=1}^{n_B} v(s_B^j) - v(z) \right) /(2t).$$

and platform A has the following profit:

$$\Pi_A = \left(P_A + \textstyle\sum_{l=1}^{k_A} \pi_A(s_A^l) \right) Q_A + \pi(z) Q_B \tag{18}$$

Solving the new system of equations leads to the following result.

Proposition 8 *A platform always finds it profitable to sell its own complementary products to consumers in the rival platform, and charges a price z ($> s^M$) that solves:*

$$L(z) = \frac{1}{1-Q_A} \frac{1}{\varepsilon(z)}$$

The rival platform always has an interest in selling these complementary products to its consumers.

When a platform has the option to sell its own complementary products to consumers of the rival platform, it should always choose to do so, although only at prices that are strictly above the monopoly level. Indeed, we obtain the familiar price-cost markup formula adjusted by market share: as a platform gets a more dominant position, it becomes less valuable to sell to consumers in the rival group since there are fewer of them, and therefore it is optimal to charge higher prices. Interestingly, the rival group also benefits from this possibility. Indeed, the platform becomes more attractive to consumers, which allows all the firms in the group to charge higher prices, obtain a bigger market share, and earn more profits.

6 Exclusivity and compatibility

6.1 Contractual exclusivity

Suppose that all complementors are, in principle, compatible with both platforms. In the absence of technological barriers, platforms may still secure alliances with other firms by signing exclusivity contracts with complementors. Under such contracts, the platform ensures incompatibility or unavailability of the complementor to the other platform. The last generation of DVD formats is a well-known example (see the discussion presented in the introduction). The ultimate objective of this strategy was to tip the market, and it eventually succeeded. However, interesting insights can be gained even when both platforms survive.

Suppose that all complementors are in principle compatible with both platforms but, due to the firms' characteristics and history of interactions, platform h has the possibility to sign an exclusivity agreement with the n_h complementors in group h. The agreement prescribes that those complementors will deliver products only for platform h. Assume (without loss of generality as we will see below) that these contracts are signed with all or none of the n_h complementors. If both platforms sign exclusive agreements with their complementors, the problem reduces to that analyzed in **[C]**. The vector of equilibrium prices is $(s_A^*, s_B^*, P_A^*, P_B^*)$, and the profits of platforms are $\Pi_A^* = (P_A^*)^2/2t$ and $\Pi_B^* = (P_B^*)^2/2t$. If no platform signs an exclusive deal, all complementors are available for all platforms. Complementor i ($\in \{1, ..., n\}$) chooses two prices, s_A^i and s_B^i, one for each group of consumers. These prices solve the following maximization problem:

$$\begin{aligned} \max_{s_A^i, s_B^i} \quad & \pi(s_A^i)Q_A + \pi(s_B^i)Q_B \\ \text{s.t.} \quad & Q_A + Q_B = 1 \end{aligned}$$

and the solution is $s_A^i = s_B^i = s^M$. Intuitively, under full market coverage and additive separability of consumers utility, complementors do not compete with each other. Each complementor realizes that its price affects the fraction of consumers who purchase their good from one platform or the other but not the total sales. It is then optimal to charge monopoly price to every consumer. In equilibrium, the marginal consumer is:

$$\bar{x} = \frac{t - P_A + P_B}{2t}$$

and the equilibrium prices and profits of platforms are $\check{P}_A = t$, $\check{P}_B = t$, $\check{\Pi}_A = (\check{P}_A)^2/2t$ and $\check{\Pi}_B = (\check{P}_B)^2/2t$.

Finally, suppose that only firms in group A sign the exclusivity agreement. The n_B complementors in B produce for both platforms so they charge monopoly prices to every consumer. Complementors in A only produce for platform A. Using a similar optimization problem as in section 3, the prices of the other players, (P_A^e, P_B^e, s_A^e), are given by:

$$\begin{aligned} \phi(s_A^e) &= t + \frac{1}{3}\left[n_A\, v(s_A^e)\right] \\ P_A^e &= \phi(s_A^e) \\ P_B^e &= t - \frac{1}{3}\left[n_A\, v(s_A^e)\right] \end{aligned}$$

with profits taking the familiar expression $\Pi_A^e = (P_A^e)^2/2t$ and $\Pi_B^e = (P_B^e)^2/2t$. Comparing this solution to full compatibility, it is immediate that $\Pi_A^e > \check{\Pi}_A$: if B chooses non-exclusivity, A gains by signing exclusive contracts with its complementors.

Now, consider the case where A signs exclusive contracts and let's look at the incentives of B. Under two-sided exclusivity, prices are summarized by system **[C]**. If B does not sign exclusive contracts, its price is $P_B^e < P_B^*$, which results in a reduction of equilibrium profits. Combining both cases, we obtain the following result.

Proposition 9 *Each platform has individual incentives to sign exclusive agreements with all the complementors in its group.*

Exclusivity is a dominant strategy for both firms and therefore the unique equilibrium for a rather simple reason. If A signs exclusive contracts with the complementors in its group, B loses the advantage of having these n_A firms providing goods for its platform. If B does not sign exclusive agreements with the complementors in its group, A keeps the advantage of having n_B complementors in both groups. Notice that each platform would ideally prefer to be the only one signing exclusive contracts. In equilibrium, however, the platform with more complementors is better-off under two-sided exclusivity than under full compatibility whereas the platform with fewer complementors is worse-off. Naturally, and as discussed in Proposition 8, exclusivity would not be optimal anymore if the platform could control the prices and receive the proceeds of the complementors.

Proposition 9 shows that each platform has individual incentives to sign exclusive dealing contracts with the complementors in its group. In industries where the platform controls the technology necessary to develop the complementary goods (information, code, etc.), it may be reasonable to assume that such arrangements can be imposed. In other industries, however, this will not be feasible and the platform will be required to compensate complementors for signing exclusive contracts. Exclusivity will then emerge only if it results in higher profits for the group. The question is whether there exist situations under which such agreement is jointly profitable. In Appendix A10, we argue that the answer depends on the market conditions. In particular, we show that with a symmetric number of complementors ($n_A = n_B = n$) and a constant elasticity of demand ($q(s) = s^{-2}$), there exists a threshold $\gamma^*(n)$ such that a group prefers exclusive contracts if $ct > \gamma^*(n)$ and non-exclusive contracts if $ct < \gamma^*(n)$, provided that the other group chooses exclusive contracts. Moreover, simulations suggest that $\gamma^*(n)$ increases with n. The intuition is simple. Other things equal, a larger degree of product differentiation, t, implies less competition between the groups, and therefore more profits available to the platform that can be used to compensate complementors. The cost of such compensation is higher as the number of complementors increases.

6.2 Welfare implications of compatibility and exclusivity

A welfare analysis can provide further insights on the merits of compatibility vs. exclusive dealing in markets with competing platforms. In this section, we compare aggregate consumer surplus as well as total surplus (aggregate consumer surplus plus profits of all firms) under two regimes: full compatibility and exclusivity.

Consumers in our model face the following trade-offs. Compared with full compatibility, exclusivity implies that consumers can purchase fewer complementary goods but at lower prices. Moreover, depending on the relative size of the number of complementors, the market shares and platform prices under the two regimes differ. This in turns implies that consumers' traveling costs associated with platform differentiation vary across the two regimes.

To gain some insights, we consider the symmetric situation $n_A = n_B$. In this case, the equilibrium market share of each platform under the two regimes is identical and equal to one-half. This implies that each consumer incurs the same traveling cost and pays the same platform price under the two regimes. Furthermore, each consumer enjoys exactly twice the number of complementary goods under full compatibility than under exclusivity. Thus, full compatibility is preferred by consumers to exclusivity if and only if the consumer surplus of each complementary good at the lower equilibrium price is smaller than twice the consumer surplus at the higher monopoly price, that is, if:

$$v(\tilde{\tilde{s}}) < 2\, v(s^M), \tag{19}$$

where $\tilde{\tilde{s}} = \phi^{-1}(t)$ is the equilibrium price for each complementary good under exclusivity.

The validity of this inequality depends on the demand function, $q(s)$, and the product differentiation parameter, t. For instance, if the demand function is linear or exponential then (19) holds for all t. If the demand function has constant elasticity, then the inequality holds generically when t is sufficiently large. A comparison of total surplus yields a similar outcome. We summarize the result in the following proposition.

Proposition 10 *Suppose $n_A = n_B$. There exist t_1 and t_2 with $t_1 \geq t_2 \geq 0$ such that:*

(i) the aggregate consumer surplus under full-compatibility exceeds that under exclusivity if and only if $t > t_1$; and

(ii) the total surplus under full-compatibility exceeds that under exclusivity if and only if $t > t_2$.

Proposition 10 implies that in the case of a symmetric number of complementors, whenever full compatibility generates higher consumer surplus than exclusivity, the same ranking also works for total surplus.

When the number of complementors associated with each platform is different, a comparison of consumer surplus and total surplus under the two regimes involves some other effects as well. We consider an extreme case in which n_A is sufficiently larger than n_B, so that the inequality $n_A \geq n_B\, v(c)/v(\phi^{-1}(2t))$ is satisfied. Following Proposition 1(ii), this implies that platform A has the whole market and that the prices of firms are given by $(\tilde{P}_A, \tilde{P}_B, \tilde{s}_A, \tilde{s}_B)$. In this equilibrium, the surplus of a consumer located at x is $n_B v(c) + t - tx$, which is independent of n_A . Since consumers are uniformly distributed on $[0, 1]$, the aggregate consumer surplus is $n_B v(c) + t/2$. On the other hand, the aggregate consumer surplus under full compatibility is $(n_A + n_B)v(s^M) - 5t/4$. Therefore, consumers are generally better-off with full compatibility when n_A is sufficiently large. We conjecture that this ranking of consumer surplus also holds in an interior equilibrium when n_A is sufficiently larger than n_B.

7 Discussions

In this paper, we have provided a model to examine the role of complementors in platform competition. The main innovative feature of the paper has been to explicitly model the three-way interaction between 2 platforms, n complementors and a continuum of consumers. The starting point and focus of the analysis is the presence of indirect externalities. Generally speaking, an increase in the number of complementors and/or the quality of a platform increases the desirability of the entire group and therefore affects the choices and profits of firms in the other group as well. Although technological differences between platforms are obviously important, these coalitions directed to promote certain standards have played a crucial role in techonological wars, most notably in industries such as high definition DVD players, music and video streaming, videogame consoles, and operating systems for computers and smartphones.

Our analysis has offered some insights on a variety of issues: (a) the optimal prices charged to consumers by competing platforms; (b) the incentives of platforms to charge royalties or offer subsidies to complementors in their group; (c) the benefits for a platform of selling its complementary products to consumers in the rival group; and (d) the incentives to form alliances through contractual exclusivity and its welfare implications. One strength of this model is its tractability. Many important elements that have been left out could, in principle, be incorporated in the analysis without having to change the basic structure of the model. We conclude with a brief discussion of some possible extensions.

Consumers' choice. In the paper, platforms compete for a fixed number of consumers of mass 1, and full market coverage is ensured by a sufficiently low transportation cost (Assumption 2). One could extend the consumer space to $x \in [\alpha, \beta]$ with $\alpha < 0$ and $\beta > 1$. Under this alternative specification, prices and transportation cost would still affect the location of the consumer $x' \in (0, 1)$ indifferent between buying from either group. However, they would also affect the location of the consumer $x'' \in [\alpha, 0)$ indifferent between buying from group A and not buying as well as the location of the consumer $x''' \in (1, \beta]$ indifferent between buying from group B and not buying. In other words, these additional parameters would extend the market coverage analysis and the incentives to capture extra consumers through prices.

One could also explicitly consider the possibility that consumers adopt multiple, competing platforms. A consumer who chooses to "multi-home" has access to all complementors but also incurs both transportation costs. Multi-homing introduces an extra dimension in the characterization of the equilibrium prices since demand curves are kinked. This, in turn, implies that for some parameter configurations multiple equilibria may arise.

Complementors' choice. While we have briefly considered the endogenous determination of complementors in a free-entry equilibrium with technological cost of adaptation, we have imposed that complementors can only produce for one platform. In a future research, we could extend the analysis and consider the possibility of producing for either or both platforms (single-production vs. multi-production). Analyzing the endogenous interaction between the frequency of multi-homing by consumers and the frequency of multi-production by complementors (e.g., as a function of the degree of differentiation in consumers' taste and the degree of differentiation in the producers' technological cost of adaptation) could also be of interest. We conjecture a negative relationship between the two. As more consumers decide to multi-home, complementors have fewer incentives to produce goods for both platforms, since they can reach a higher fraction of consumers through either of them. Conversely, as more complementors choose to multi-produce, consumers have fewer incentives to purchase both platforms, since most goods are available in each platform.

Technological compatibility. We have discussed contractual agreements that restrict the diffusion of products among the different platforms. It would be interesting to study technological compatibility, which can in fact be seen as the flip side problem. Indeed, it is possible that due to technological reasons complementors designed for one platform cannot operate in the other group. Platforms, however, may invest resources in achieving compatibility with the complementors in the other platform. This would correspond to an outcome of complete compatibility without standardization, as each provider still keeps a differentiated platform, that is, a different location in the Hotelling line. The current hardware complementors linked to personal computers (printers, cameras, speaker-phones) is a good example, as they can be connected to all operating systems through universal USB ports.

Although the cases of compatibility and exclusivity are formally rather similar, the results may not be. Each platform individually gains with technological compatibility (as it attracts the complementors in the other group) and with contractual exclusivity (as it prevents its own complementors to sell to the other group). We therefore expect full compatibility to be the outcome in the technological compatibility game whereas two-sided exclusivity to be the outcome in the contractual exclusivity game.

Standardization. Our framework could also be applied to study players' decision to standardize their platforms. We view *complete standardization* as a situation in which platform providers agree to choose a single platform, i.e. a single location in the Hotelling setting, and share the revenues from royalties and platform sales. CDs and past generation of DVDs are good examples. This is not the same as incompatibility and competition where only one platform survives (the corner solution case in section 3), which has occurred for example when Blu-ray drove HD-DVD out of the market. The outcome is identical, but there are important differences: with standardization (i) single platform arises from an ex-ante agreement among providers rather than from an ex-post market competition outcome, and (ii) the location in the Hotelling line is likely to be closer to the interests of the median consumer. Complete standardization is also different from the complete compatibility without standardization setting discussed above, where platforms choose different locations and agree to make all complementors compatible with each other. As developed above, with multiple locations, there is price competition between platforms and a shorter traveling distance for the average consumer.

Integration. The analysis in section 5 assumes that the ownership of complementors (by the platform or by independent firms) is exogenously given. A natural extension would be to consider the incentives of platforms to integrate some or all the complementors in their group. From the previous analysis, we know that integration has three direct effects in the profits of a group: the platform and independent complementors are sold at higher prices yielding higher profits, but the newly integrated complementors are sold at no profit. Furthermore, the integration strategy in one group affects the pricing and integration decision in the other. A full characterization of the equilibrium of the integration game is an interesting challenge for future research.

Other applications. Although initially motivated by IT industries, our framework applies also to more traditional market settings. For example, golf and tennis clubs can be thought of as platforms that offer valuable complementary services to their members: golf and tennis lessons, pool, gym, shopping and restaurants. Some of these services are integrated (typically, pool and gym) whereas others are outsourced (typically, pro-shop and restaurant). Shopping malls and recreational parks are also platforms that provide a variety of complementary products. The former compete for consumers by proposing retailers, restaurants, movie theaters and other entertainment options whereas the latter compete for visitors by offering rides, shows and organized group events. Finally, even religions could, from a pure economic viewpoint, be formalized as horizontally differentiated platforms that offer complementary services to their members.

Appendix

A0. The following lemma is useful in the proof of Propositions 1 and 2 and its proof involves straightforward calculations.

Lemma A0: *Let (s_A^*, s_B^*) satisfy the first two equations in [C]. The following holds:*

$$K\frac{\partial s_A^*}{\partial t} = 3\phi'(s_B^*) + 2n_B q(s_B^*), \quad K\frac{\partial s_A^*}{\partial w_A} = \phi'(s_B^*), \quad K\frac{\partial s_A^*}{\partial w_B} = -\phi'(s_B^*),$$

$$K\frac{\partial s_A^*}{\partial n_A} = v(s_A^*)\phi'(s_B^*), \quad K\frac{\partial s_A^*}{\partial n_B} = -v(s_B^*)\phi'(s_B^*)$$

and symmetrically for s_B^*, where $K \equiv 3\phi'(s_A^*)\phi'(s_B^*) + n_A q(s_A^*)\phi'(s_B^*) + n_B q(s_B^*)\phi'(s_A^*)$.

A1. Proof of Proposition 1

Part (i). As discussed in the text, the sufficient conditions for the solution to **[C]** to be an equilibrium are satisfied. Assumption 1 and the equilibrium conditions (9) and (10) imply that equilibrium prices among complementors in each group are identical. We only need to show that **[C]** admits a unique interior solution. Clearly it suffices to show that the first two equations in **[C]** have a unique interior solution. Let $s_B = s$ and write the two equations as follows

$$\begin{aligned} \phi(x(s)) &= t + \frac{1}{3}\Big(n_A v(x(s)) - n_B v(s) + \Delta w\Big), \\ \phi(s) &= t - \frac{1}{3}\Big(n_A v(y(s)) - n_B v(s) + \Delta w\Big), \end{aligned}$$

which uniquely determine two functions $x(s) \in (c, s^M)$ and $y(s) \in (c, s^M)$ for some range of $s \in (c, s^M)$, respectively. To obtain a unique interior solution, it then suffices to show that there exists a unique $s^* \in (c, s^M)$ such that $x(s^*) = y(s^*) \in (c, s^M)$.

First note that when $x(s) \in (c, s^M)$ and $y(s) \in (c, s^M)$,

$$x'(s) = \frac{n_B q(s)}{3\phi'(x(s)) + n_A q(x(s))}, \quad y'(s) = \frac{3\phi'(s) + n_B q(s)}{n_A q(y(s))},$$

implying that $x'(s) > 0$ and $y'(s) > 0$ since $\phi'(s) > 0$ for $s \in (c, s^M)$. Moreover, $y'(s) > x'(s)$ whenever $x(s) = y(s)$, implying that the two curves cannot intersect more than once.

We now show that under the assumptions in Part (i) of Proposition 1, the two curves indeed intersect. Since $n_A v(c) + w_A \geq n_B v(c) + w_B$ and $\lim_{s \to s^M} \phi(s) = +\infty$, it is immediate that $x(c) > c$ and $x(s^M) < s^M$, and that there exists $\hat{s} \in (c, s^M)$ such that $y(\hat{s}) = s^M$.

Next, consider two cases. If $3t \geq n_A v(c) - n_B v(c) + \Delta w$, then there exists $s_0 \in (c, s^M)$ such that $y(s_0) = c$, implying that $y(s_0) < x(s_0)$. It follows from the continuity of $x(s)$ and $y(s)$ and the Intermediate Value Theorem that the two curves intersect exactly once in $(s_0, \hat{s})$. If $3t < n_A v(c) - n_B v(c) + \Delta w$, then

$$y(c) = v^{-1}\left(\frac{3t + n_B v(c) - \Delta w}{n_A}\right) > c$$

and $y(c)$ decreases with t. On the other hand, $x(c)$ increases with t. Note that $x(c) = y(c) = z$ whenever

$$\begin{aligned} \phi(z) &= t + \frac{1}{3}\Big(n_A v(z) - n_B v(c) + \Delta w\Big), \\ 0 &= t - \frac{1}{3}\Big(n_A v(z) - n_B v(c) + \Delta w\Big), \end{aligned}$$

or equivalently $\phi(z) = 2t$. That is, $x(c) = y(c)$ when t satisfies

$$v^{-1}\left(\frac{3t + n_B v(c) - \Delta w}{n_A}\right) = \phi^{-1}(2t),$$

or $n_A\, v(\phi^{-1}(2t)) + w_A = n_B\, v(c) + w_B + 3t$. It follows that $y(c) < x(c)$ when

$$n_A\, v(\phi^{-1}(2t)) + w_A < n_B\, v(c) + w_B + 3t.$$

Again, in this case, by the continuity of $x(s)$ and $y(s)$ and the Intermediate Value Theorem the two curves intersect once in $(c, \hat{s})$. Therefore, the interior solution is unique.

We next show the condition for full market coverage at the interior solution is satisfied. Let

$$G(n_A, n_B, w_A, w_B) \equiv n_A v(s_A^*) + w_A + n_B v(s_B^*) + w_B - 3t.$$

Then $U_A(\bar{x}) = U_B(\bar{x}) = G(n_A, n_B, w_A, w_B)/2$ and the condition for full market coverage is $G \geq 0$. Note that if $n_A = n_B = 1$ and $w_A = w_B = 0$ then $s_A^* = s_B^* = \phi^{-1}(t)$ and hence by Assumption 2, $G(1, 1, 0, 0) = 2v(\phi^{-1}(t)) - 3t \geq 0$. Note also by Lemma A0 that

$$\frac{\partial G}{\partial n_A} > 0, \quad \frac{\partial G}{\partial n_B} > 0, \quad \frac{\partial G}{\partial w_A} > 0, \text{ and } \frac{\partial G}{\partial w_B} > 0.$$

It follows that $G(n_A, n_B, w_A, w_B) \geq 0$ for any $n_A \geq 1$, $n_B \geq 1$, $w_A \geq 0$, and $w_B \geq 0$.

We now prove a slightly more general version of Part (ii), by finding all the possible corner equilibria. Suppose $s_A^{i'} = \hat{s}_A$ for all $i' \neq i$. Also, $P_B = 0$, and $s_B^j = c$ for all j. Using (5), the market share for group A is

$$\bar{x} = \frac{t + \Delta w - P_A + (n_A - 1)v(\hat{s}_A) + v(s_A) - n_B v(c)}{2t}.$$

The pairs $(\hat{s}_A, \hat{P}_A)$ for complementor i and platform A such that $\bar{x} = 1$ are given by

$$\bar{x} = 1 \quad \Leftrightarrow \quad n_A v(\hat{s}_A) - \hat{P}_A + w_A = n_B v(c) + w_B + t.$$

Given $\hat{P}_A$, complementor i chooses s_A^i to maximize $\pi(s_A^i)\bar{x}$. We have

$$\left.\tfrac{d\pi(s_A^i)\bar{x}}{ds_A^i}\right|_{s_A^i = \hat{s}_A^-} = \pi'(s_A^i) > 0 \;\text{ and }\; \left.\tfrac{d\pi(s_A^i)\bar{x}}{ds_A^i}\right|_{s_A^i = \hat{s}_A^+} = \tfrac{\pi'(s_A^i)}{2t}\left(2t - \phi(s_A^i)\right) \leq 0 \;\text{ iff }\; \phi(s_A^i) \geq 2t.$$

Analogously, given $\hat{s}_A$, platform A chooses P_A to maximize $P_A\bar{x}$. We have

$$\left.\tfrac{dP_A\bar{x}}{dP_A}\right|_{P_A = \hat{P}_A^-} = 1 > 0 \;\text{ and }\; \left.\tfrac{dP_A\bar{x}}{dP_A}\right|_{P_A = \hat{P}_A^+} = 1 - \tfrac{P_A}{2t} \leq 0 \;\text{ iff }\; P_A \geq 2t.$$

The equilibrium thus requires

$$\phi(\hat{s}_A) \geq 2t, \quad \hat{P}_A \geq 2t, \quad n_A v(\hat{s}_A) - \hat{P}_A + w_A = n_B v(c) + w_B + t.$$

The condition for the existence of such equilibrium is

$$n_A v(\phi^{-1}(2t)) - n_B v(c) + \Delta w - 3t \geq 0. \tag{20}$$

We can then characterize the set of all possible equilibria. Formally and given (20), there will exist $\bar{k}$ defined by $n_A v(\phi^{-1}(2t + \bar{k})) - n_B v(c) + \Delta w - 3t = 0$ such that a corner equilibrium implies prices:

$$s_A = \phi^{-1}(2t + k) \;\text{ and }\; P_A = n_A v(\phi^{-1}(2t + k)) - n_B v(c) + \Delta w - t \;\text{ with }\; k \in [0, \bar{k}].$$

The equilibrium described in Proposition 1 corresponds to $k = 0$. Last, it is straightforward to verify that platform B and its complementors do not have incentives to deviate from the proposed equilibrium.

A2. Proof of Proposition 2

Part (i) follows immediately from Lemma A0 and the implication of Assumption 1.

To prove Part (ii), consider a consumer $x \le \bar{x}^*$. Its utility at the interior equilibrium is given by

$$\begin{aligned} U_A(x) &= w_A + n_A v(s_A^*) - P_A^* - tx \\ &= \frac{2}{3}w_A + \frac{1}{3}w_B + \frac{2}{3}n_A v(s_A^*) + \frac{1}{3}n_B v(s_B^*) - t(1+x). \end{aligned}$$

Utilizing Lemma A0 yields

$$K\frac{\partial U_A(x)}{\partial n_A} = v(s_A^*)\phi'(s_A^*)[2\phi'(s_B^*) + n_B q(s_B^*)] > 0.$$

Similarly,

$$K\frac{\partial U_A(x)}{\partial n_B} = v(s_B^*)\phi'(s_B^*)[\phi'(s_A^*) + n_A q(s_A^*)] > 0.$$

The other properties of Part (ii) can be shown analogously.

A3. Proof of Proposition 3

In absence of group B, we assume that platform A is still located at $x = 0$. Denote by x_A the consumer indifferent between not buying and buying from group A. Using (1), we obtain

$$x_A = \frac{w_A + \sum_i v(s_A^i) - P_A}{t}. \tag{21}$$

Platform A and complementor k in group A solve the following maximization problems, respectively,

$$\max_{P_A} \; P_A \, x_A \qquad \text{and} \qquad \max_{s_A^k} \; \pi(s_A^k)\, x_A.$$

Taking the first-order conditions in the above maximization problems and then imposing symmetry on prices of complementors, we obtain an interior equilibrium, (s_A^o, P_A^o), satisfying

$$\begin{aligned} \phi(s_A^o) &= \frac{n_A v(s_A^o) + w_A}{2}, \\ P_A^o &= \phi(s_A^o), \end{aligned}$$

which yields equilibrium market share and profits

$$x_A^o = \frac{\phi(s_A^o)}{t}, \quad \Pi_A^o = \frac{[\phi(s_A^o)]^2}{t}, \quad V_A^o = \frac{\pi(s_A^o)\phi(s_A^o)}{t}.$$

The interior equilibrium is derived under the assumption that the market is not fully covered, which occurs if

$$n_A v(\phi^{-1}(t)) + w_A < 2t. \tag{22}$$

As in part (ii) of Proposition 1, when the market is fully covered (that is, when $n_A v(\phi^{-1}(t)) + w_A \geq 2t$) there are multiple corner equilibria. In that case, the prices of the platform and complementary goods, (s_A^o, P_A^o) satisfy

$$\phi(s_A^o) \geq t, \quad P_A^o \geq t, \quad n_A v(s_A^o) - P_A^o + w_A = t.$$

We can characterize the set of all possible equilibria. Formally, there exists $\bar{l}$ defined by $n_A v(\phi^{-1}(t + \bar{l})) + w_A = 2t$ such that a full coverage equilibrium implies prices:

$$s_A = \phi^{-1}(t + l) \text{ and } P_A = n_A v(\phi^{-1}(t + l)) + w_A - t \text{ with } l \in [0, \bar{l}].$$

The equilibrium with prices $s_A^o = \phi^{-1}(t)$ and $P_A^o = n_A v(\phi^{-1}(t)) + w_A - t$ (that is, the one where $l = 0$) has the property that (i) it maximizes the total profits of the group and (ii) it is the unique equilibrium when $n_A v(\phi^{-1}(t)) + w_A = 2t$.

A direct inspection of the conditions for corner equilibrium with and without group B yields

$$n_A v(\phi^{-1}(2t)) + w_A - 2t \geq n_B v(c) + w_B + t \quad \Rightarrow \quad n_A v(\phi^{-1}(t)) + w_A - 2t \geq 0.$$

That is, if in the presence of group B, group A captures the entire market, then in the absence of group B, the entire market will remain covered by group A.

Comparing the two equilibria that maximize the total profits of group A, we see that $P_A^o > \tilde{P}_A$ and $s_A^o < \tilde{s}_A$. In each case consumers are indifferent among multiple equilibria, $U_A^o = t(1-x)$ in absence of group B and $U_A = w_B + n_B v(c) + t(1-x)$ in the presence of group B. Thus, $U_A^o < U_A$.

A4. Proof of Proposition 4

Part (i). Since $w_A = w_B$, by Proposition 1(i), if $n_A = n_B = n$ then there always exists a unique and interior pricing equilibrium in which $s_A^* = s_B^* = \phi^{-1}(t)$ and $P_A^* = P_B^* = t$, yielding the following profits for each complementor $h \in \{A, B\}$:

$$V_h^*(n, n) = \pi(\phi^{-1}(t))/2.$$

The claim follows immediately from the definition of the free-entry equilibrium.

Part (ii). Since $w_A = w_B$, by Proposition 1(ii), if $n_A v(\phi^{-1}(2t)) - 3t \geq n_B v(c)$ then there always exists corner equilibria in which all complementors in group A charge $\tilde{s}_A \geq \phi^{-1}(2t)$ and make profits as large as $\pi(\phi^{-1}(2t))$ while there is a small number of complementors in group B charging $\tilde{s}_B = c$ and making zero profits. The free-entry condition for complementors in group A becomes:

$$\pi(\phi^{-1}(2t)) \geq \delta n_A.$$

The above conditions are satisfied when

$$\delta \leq \pi(\phi^{-1}(2t))v(\phi^{-1}(2t))/[3t + n_B v(c)].$$

That is, for any given t that satisfies Assumption 2, as long as δ is smaller than or equal to the above threshold (with a small n_B), there exists a tipping equilibrium in which all complementors go with platform A that has 100% of the market share. Note that the presence of group B firms just constrains the prices and profits of firms in group A.

A5. Proof of Proposition 5

First notice that Assumption (1) implies $\phi_1(s,r) > 0$, $\phi_2(s,r) = q(s)[q'(s)\phi(s,r) - q(s)]/\pi_1(s,r) < 0$, $\rho_1(s,r) = -q(s) + rq'(s) < 0$ for $r > 0$ and $\rho_2(s,r) = q(s) > 0$. We can use system **[C']** to derive $\partial s_h/\partial r_{h'}$. Let

$$H = \begin{pmatrix} \phi_1(s_A, r_A) - \frac{1}{3}n_A\,\rho_1(s_A, r_A) & \frac{1}{3}n_B\,\rho_1(s_B, r_B) \\ \frac{1}{3}n_A\rho_1(s_A, r_A) & \phi_1(s_B, r_B) - \frac{1}{3}n_B\rho_1(s_B, r_B) \end{pmatrix}.$$

We have:

$$|H| = \phi_1(s_A, r_A)\phi_1(s_B, r_B) - \frac{1}{3}n_A\rho_1(s_A, r_A)\phi_1(s_B, r_B) - \frac{1}{3}n_B\phi_1(s_A, r_A)\rho_1(s_B, r_B) > 0.$$

Therefore:

$$\begin{aligned} \frac{\partial s_A}{\partial r_A}|H| &= \begin{vmatrix} -\phi_2(s_A, r_A) + \frac{1}{3}n_A\rho_2(s_A, r_A) & \frac{1}{3}n_B\rho_1(s_B, r_B) \\ -\frac{1}{3}n_A\rho_2(s_A, r_A) & \phi_1(s_B, r_B) - \frac{1}{3}n_B\rho_1(s_B, r_B) \end{vmatrix} \\ &= -\phi_2(s_A, r_A)\phi_1(s_B, r_B) + \frac{1}{3}n_A\rho_2(s_A, r_A)\phi_1(s_B, r_B) + \frac{1}{3}n_B\phi_2(s_A, r_A)\rho_1(s_B, r_B) \\ &> 0. \end{aligned}$$

It follows that $\dfrac{\partial s_A}{\partial r_A} > 0$. Similarly,

$$\begin{aligned} \frac{\partial s_B}{\partial r_A}|H| &= \begin{vmatrix} \phi_1(s_A, r_A) - \frac{1}{3}n_A\rho_1(s_A, r_A) & -\phi_2(s_A, r_A) + \frac{1}{3}n_A\rho_2(s_A, r_A) \\ \frac{1}{3}n_A\rho_1(s_A, r_A) & -\frac{1}{3}n_A\rho_2(s_A, r_A) \end{vmatrix} \\ &= \frac{1}{3}n_A\Big[\phi_2(s_A, r_A)\rho_1(s_B, r_B) - \rho_2(s_A, r_A)\phi_1(s_B, r_B)\Big]. \end{aligned}$$

The expression for $\frac{\partial s_A}{\partial r_B}$ and $\frac{\partial s_B}{\partial r_B}$ can be derived similarly.

Inserting these expressions in the first-order conditions (12) and (13), we find that the first-order conditions are equivalent to

$$\frac{\rho_1(s_A^{**}, r_A)}{\rho_2(s_A^{**}, r_A)} = \frac{\phi_1(s_A^{**}, r_A)}{\phi_2(s_A^{**}, r_A)},$$

$$\frac{\rho_1(s_B^{**}, r_B)}{\rho_2(s_B^{**}, r_B)} = \frac{\phi_1(s_B^{**}, r_B)}{\phi_2(s_B^{**}, r_B)}.$$

Moreover, it can be shown that, for any pair of (s, r),

$$\frac{\rho_1(s, r)}{\rho_2(s, r)} - \frac{\phi_1(s, r)}{\phi_2(s, r)} = -\varepsilon(s)L(s) + [\varepsilon(s)]^2[(s - r - c)/s]^2/\eta(s),$$

which leads to (15).

Positive royalty or subsidy follows immediately from (15).

A6. Proof of Proposition 6

Part (i). Taking first-order conditions in (17), we get that an interior price equilibrium is given by:

$$\frac{\partial \Pi_A}{\partial P_A} = 0 \quad \Rightarrow \quad Q_A - \left(P_A + \sum \pi(s_A^l)\right)/(2t) = 0 \tag{23}$$

$$\frac{\partial \Pi_A}{\partial s_A^l} = 0 \quad \Rightarrow \quad \left(q(s_A^l) + (s_A^l - c)q'(s_A^l)\right)Q_A - \left(P_A + \sum \pi(s_A^l)\right)q(s_A^l)/(2t) = 0$$

$$\Rightarrow \quad \frac{\partial \Pi_A}{\partial P_A} q(s_A^l) + (s_A^l - c)q'(s_A^l)Q_A = 0. \tag{24}$$

From (23) and (24), it is immediate that $s_A^l = c$ for all $l \in \{1, ..., k_A\}$. The same is true for platform B.

Part (ii). Following a similar reasoning as in section (3.2), we obtain that in an interior equilibrium the prices of platforms and of the independent complementors solve the following system of equations [**D**]:

$$\mathbf{D}: \quad \begin{aligned} \phi(s_A) &= t + \frac{1}{3}\Big[k_A v(c) + (n_A - k_A)v(s_A)\Big] - \frac{1}{3}\Big[k_B v(c) + (n_B - k_B)v(s_B)\Big] \\ \phi(s_B) &= t - \frac{1}{3}\Big[k_A v(c) + (n_A - k_A)v(s_A)\Big] + \frac{1}{3}\Big[k_B v(c) + (n_B - k_B)v(s_B)\Big] \\ P_A &= \phi(s_A) \\ P_B &= \phi(s_B) \end{aligned}$$

Differentiating the first two equilibrium conditions, we get:

$$\begin{aligned}\Big(\phi'(s_A)+\frac{1}{3}(n_A-k_A)q(s_A)\Big)\frac{\partial s_A}{\partial k_A}-\frac{1}{3}(n_B-k_B)q(s_B)\frac{\partial s_B}{\partial k_A} &= \frac{1}{3}(v(c)-v(s_A))\\ \phi'(s_A)\frac{\partial s_A}{\partial k_A}+\phi'(s_B)\frac{\partial s_B}{\partial k_A} &= 0.\end{aligned}$$

It follows that:

$$\Big(3\phi'(s_A)\phi'(s_B)+(n_A-k_A)q(s_A)\phi'(s_B)+(n_B-k_B)q(s_B)\phi'(s_A)\Big)\frac{\partial s_A}{\partial k_A}=v(c)-v(s_A).$$

Thus, $\partial s_A/\partial k_A>0$ and $\partial s_B/\partial k_A<0$. Since $P_h=\phi(s_h)$, $Q_h=\phi(s_h)/2t$, $\Pi_h=\phi(s_h)^2/2t$, $V_h=\pi(s_h)\phi(s_h)/2t$, this in turn implies that $\partial P_A/\partial k_A>0$, $\partial P_B/\partial k_A<0$, $\partial Q_A/\partial k_A>0$, $\partial Q_B/\partial k_A<0$, $\partial \Pi_A/\partial k_A>0$, $\partial \Pi_B/\partial k_A<0$, $\partial V_A/\partial k_A>0$, $\partial V_B/\partial k_A<0$. A symmetric argument holds for B.

A7. Proof of Proposition 7

Let $n_A>n_B$, so that group A is the more valuable ($n_A=n_B$ is simply the limit case). The interior equilibrium is the solution of system **[C]**, denoted by $(P_A^*,P_B^*,s_A^*,s_B^*)$. Group A's market share is $Q_A^*=P_A^*/2t=\phi(s_A^*)/2t$.

Under full integration, the price of complementors is $s_A^I=s_B^I=c$ (see Proposition 6). Setting $k_A=n_A$ and $k_B=n_B$ in system **[D]**, we obtain:

$$P_A^I=t+\frac{1}{3}(n_A-n_B)v(c),\quad P_B^I=t-\frac{1}{3}(n_A-n_B)v(c),\quad Q_A^I=\frac{P_A^I}{2t}$$

Comparing market shares under the two regimes we get:

$$Q_A^I-Q_A^*\;\propto\;n_A\Big(v(c)-v(s_A^*)\Big)-n_B\Big(v(c)-v(s_B^*)\Big)$$

Finally, $n_A>n_B\;\Rightarrow\;s_A^*>s_B^*\;$ and $\;v(s_A^*)<v(s_B^*)\;\Rightarrow\;Q_A^I>Q_A^*$.

To evaluate consumer's welfare, let $U_h^*(x)$ and $U_h^I(x)$ be the utility of a consumer located at x who buys from group h under no integration and full integration, respectively.

For $x\in[0,Q_A^*]$ (consumers who buy from group A both under full and no integration), we have:

$$U_A^I(x)-U_A^*(x)=\frac{2}{3}n_A\Big(v(c)-v(s_A^*)\Big)+\frac{1}{3}n_B\Big(v(c)-v(s_B^*)\Big)>0$$

For $x\in[Q_A^I,1]$ (consumers who buy from group B both under full and no integration), we have:

$$U_B^I(x)-U_B^*(x)=\frac{1}{3}n_A\Big(v(c)-v(s_A^*)\Big)+\frac{2}{3}n_B\Big(v(c)-v(s_B^*)\Big)>0$$

For $x \in [Q_A^*, Q_A^I]$ (consumers who switch from group B under no integration to group A under full integration), we have:

$$U_A^I(x) - U_B^*(x) > U_B^I(x) - U_B^*(x) = \frac{1}{3}n_A\Big(v(c) - v(s_A^*)\Big) + \frac{2}{3}n_B\Big(v(c) - v(s_B^*)\Big) > 0.$$

which proves that all consumers are better-off under full integration.

A8. Proof of Proposition 8

Taking first-order conditions in (18) and noting that $Q_B = 1 - Q_A$, we get that an interior price equilibrium is given by:

$$\frac{\partial \Pi_A}{\partial P_A} = 0 \quad \Rightarrow \quad Q_A - \Big(P_A + \sum \pi(s_A^l) - \pi(z)\Big)/(2t) = 0 \tag{25}$$

$$\frac{\partial \Pi_A}{\partial s_A^l} = 0 \quad \Rightarrow \quad \frac{\partial \Pi_A}{\partial P_A} q(s_A^l) + (s_A^l - c)q'(s_A^l)Q_A = 0 \tag{26}$$

$$\frac{\partial \Pi_A}{\partial z} = 0 \quad \Rightarrow \quad \left(Q_A - \frac{\partial \Pi_A}{\partial P_A}\right) q(z) + \pi'(z)(1 - Q_A) = 0 \tag{27}$$

Combining (25) and (26) we get, just as before, $s_A^l = c$. Combining (25) and (27) we get the price formula of z. The other prices solve the following system of equations:

$$\begin{aligned}
\mathbf{D'}: \quad \phi(s_A) &= t + \frac{1}{3}\Big[k_A v(c) + (n_A - k_A)v(s_A)\Big] - \frac{1}{3}\Big[k_B v(c) + (n_B - k_B)v(s_B)\Big] - \frac{1}{3}v(z) \\
\phi(s_B) &= t - \frac{1}{3}\Big[k_A v(c) + (n_A - k_A)v(s_A)\Big] + \frac{1}{3}\Big[k_B v(c) + (n_B - k_B)v(s_B)\Big] + \frac{1}{3}v(z) \\
P_A &= \phi(s_A) \\
P_B &= \phi(s_B)
\end{aligned}$$

Differentiating the equations, we get $ds_B/dz < 0$ and $dP_B/dz < 0$. Thus, relative to system **[D]** where there is no $v(z)$, the solution to system **[D']** implies higher prices s_B and P_B, and therefore higher profits Π_B and V_B.

A9. Proof of Proposition 9

Immediate, thus omitted.

A10. Net benefits of exclusive dealing for the group

Let $q(s) = s^{-2}$. This means that $v(s) = s^{-1}$, $\pi(s^M) = \frac{1}{4c}$, and $\phi(s) = \frac{s-c}{s(2c-s)}$. Also, $n_A = n_B = n$. We compare the profits of group B under two cases: both platforms use exclusive contracts [EE] and platform A uses exclusive contracts and platform B non-exclusive contracts [EN].

The total profits of group B under [EE] are:

$$T^{EE} = \Pi + nV = \frac{t}{2} + n\frac{\pi(s^*)}{2}$$

From the solution in section 3, we get:

$$t = \phi(s^*) \;\Rightarrow\; t = \frac{s-c}{s^*(2c-s^*)} \;\Rightarrow\; s^* = \frac{1}{2t}\left((2\gamma-1)+\sqrt{4\gamma^2+1}\right)$$

where $\gamma \equiv c \cdot t$.

The total profits of group B under [EN] are:

$$T^{EN} = \Pi_B + nV_B = \frac{1}{2t}\left(t - \frac{n \cdot v\left(s_A^*\right)}{3}\right)^2 + n\pi\left(s^M\right)$$

Again from section 3, s_A^* is such that,

$$\phi\left(s_A^*\right) = t + \frac{n \cdot v\left(s_A^*\right)}{3} \Rightarrow s_A^* = \frac{-\left(3-6\gamma+n\right)+\sqrt{36\gamma^2+12n\gamma+n^2+6n+9}}{6t}$$

Let:

$$f(\gamma) = c \cdot T^{EE} = c\frac{t}{2} + c\frac{n}{2}\frac{\left(s^*-c\right)}{(s^*)^2} = \frac{1}{2}\gamma\left(1+\frac{2n\left(-1+\sqrt{4\gamma^2+1}\right)}{\left(2\gamma-1+\sqrt{4\gamma^2+1}\right)^2}\right)$$

and

$$g(\gamma) = c \cdot T^{EN} = \frac{n}{4} + \frac{c}{2t}\left(t - \frac{n}{3 \cdot s_A^*}\right)^2 = \frac{n}{4} + \frac{\gamma}{2}\left(1 - \frac{2n}{\left[-\left(3-6\gamma+n\right)+\sqrt{36\gamma^2+12n\gamma+n^2+6n+9}\right]}\right)^2$$

We have an interior equilibrium in [EN] if:

$$s_A^* t > \frac{n}{3} \quad\Rightarrow\quad -3+6\gamma-3n+\sqrt{36\gamma^2+12n\gamma+n^2+6n+9} > 0$$

$$\Rightarrow\quad \gamma > \underline{\gamma} \equiv k(n), \text{ where } \; k(n) \equiv \frac{n\left(2n+3\right)}{3\left(4n+3\right)}$$

Also, we have full market coverage in [EE] if:

$$n \cdot v(s^*) - \frac{3}{2} \cdot t \geq 0 \quad\Rightarrow\quad \frac{4n}{(2\gamma-1)+\sqrt{4\gamma^2+1}} - 3 \geq 0$$

$$\Rightarrow\quad \gamma \leq \bar{\gamma} \equiv 2 \cdot k(n)$$

Therefore, our task is to compare $f(\gamma)$ and $g(\gamma)$ for $\gamma \in [\underline{\gamma}, \bar{\gamma}]$. Let $\Delta(\gamma) \equiv f(\gamma) - g(\gamma) + g(0)$. We first study the behavior of $\Delta(\gamma)$ for $\gamma \in \mathbb{R}_+$. The following hold:

(i) $\Delta(0) = 0$.

(ii) $\Delta(\gamma)/\gamma$ decreases with γ for all $\gamma \in \mathbb{R}_+$. Note that $(g(\gamma) - g(0))/\gamma$ is increasing with respect to γ:

$$\frac{\partial(g(\gamma) - g(0))/\gamma}{\partial\gamma} = \frac{2n}{\Lambda}\left\{\left(\frac{2n}{\Lambda} + 1\right)\left(\frac{12n + 72\gamma}{2\sqrt{n^2 + 12n\gamma} + 6n + 36\gamma^2 + 9} + 6\right)\right\} > 0 \quad (28)$$

where

$$\Lambda = 3 - 6\gamma + n - \sqrt{36\gamma^2 + 12n\gamma + n^2 + 6n + 9}\gamma$$

Similarly, note that $f(\gamma)/\gamma$ is decreasing with respect to γ since, by letting $z \equiv \sqrt{4\gamma^2 + 1}$,

$$\frac{\partial f(\gamma)/\gamma}{\partial\gamma} = n\frac{8\gamma^2 - 4\gamma[z-1] - 4z[z-1]}{\sqrt{4\gamma^2+1}\left(2\gamma + \sqrt{4\gamma^2+1} - 1\right)^3} < 0 \quad (29)$$

(iii) Equations (28) and (29) imply that $\lim_{\gamma\to 0}\frac{\Delta(\gamma)}{\gamma} = \infty$.

(iv) $\lim_{\gamma\to\infty}\Delta(\gamma) = \frac{7n}{24}$.

From (i)-(ii)-(iii)-(iv), we conclude that $f(\gamma)$ and $g(\gamma)$ cross uniquely at γ^*. Next, we show in simulations that $f(\underline{\gamma}) < g(\underline{\gamma})$ and $f(\bar{\gamma}) > g(\bar{\gamma})$, so we conclude that there is a unique $\gamma^*(n) \in (\underline{\gamma}, \bar{\gamma})$ such that group B prefers [EE] to [EN] only if $\gamma > \gamma^*(n)$. Moreover, $\gamma^*(n)$, increases with n.

A11. Proof of Proposition 10

With full compatibility (FC) or non-exclusivity, the market share for each platform is 1/2, the price of each platform is equal to t, and consumers enjoy all $n_A + n_B$ number of complementary products at a monopoly price s^M. Thus, the net surplus for a consumer in $x \in [0, 1/2]$ is

$$(n_A + n_B)v(s^M) - t - tx.$$

Similar expression can be written for consumers in $x \in (1/2, 1]$. Since consumers are uniformly distributed on $[0, 1]$, the aggregate consumer surplus under FC is then

$$CS_{FC} = (n_A + n_B)v(s^M) - 5t/4.$$

Since the profit for each platform is equal to $t/2$ and each complementor makes $\pi(s^M)$, the total surplus under FC is

$$TS_{FC} = (n_A + n_B)TS(s^M) - t/4$$

where $TS(s) = v(s) + \pi(s)$ is the total surplus of a complementary product at price s.

With non-compatibility or exclusivity (EX), the equilibrium market share for A is

$$\bar{x} = \phi(s_A)/(2t).$$

Consumers in $x \in [0, \bar{x}]$ enjoy all n_A complementary products at a price s_A:

$$U(x) = n_A v(s_A) - \phi(s_A) - tx.$$

Total consumer surplus now is equal to

$$\begin{aligned} CS_{EX} &= [n_A v(s_A) - \phi(s_A)]\bar{x} - t(\bar{x})^2/2 + [n_B v(s_B) - \phi(s_B)](1-\bar{x}) - t(1-\bar{x})^2/2 \\ &= \left(n_A v(s_A) - \frac{5}{4}\phi(s_A)\right)\frac{\phi(s_A)}{2t} + \left(n_B v(s_B) - \frac{5}{4}\phi(s_B)\right)\frac{\phi(s_B)}{2t} \end{aligned}$$

Total complementors' profits are

$$\begin{aligned} & n_A \pi(s_A)\bar{x} + n_B \pi(s_B)(1-\bar{x}) \\ = \;& n_A \pi(s_A)\frac{\phi(s_A)}{2t} + n_B \pi(s_B)\frac{\phi(s_B)}{2t}. \end{aligned}$$

Platform profits are

$$\frac{[\phi(s_A)]^2}{2t} + \frac{[\phi(s_B)]^2}{2t}.$$

Thus,

$$TS_{EX} = n_A TS(s_A)\frac{\phi(s_A)}{2t} + n_B TS(s_B)\frac{\phi(s_B)}{2t} - \frac{1}{8t}\left([\phi(s_A)]^2 + [\phi(s_B)]^2\right).$$

When $n_A = n_B = n$, $s_A = s_B = \phi^{-1}(t)$, it follows that

$$\begin{aligned} CS_{FC} &= 2nv(s^M) - 5t/4, \\ CS_{EX} &= nv(\phi^{-1}(t)) - 5t/4, \\ TS_{FC} &= 2nTS(s^M) - t/4, \\ TS_{EX} &= nTS(\phi^{-1}(t)) - t/4. \end{aligned}$$

Define $t_1 \geq 0$ such that $t_1 = 0$ if $2v(s^M) > v(c)$ and

$$2v(s^M) = v(\phi^{-1}(t_1))$$

otherwise. Similarly, define $t_2 \geq 0$ such that $t_2 = 0$ if $2TS(s^M) > TS(c)$ and

$$2TS(s^M) = TS(\phi^{-1}(t_2))$$

otherwise. The claims then follow from the fact that $v(s)$ is strictly decreasing and $\phi(s)$ is strictly increasing.

Finally, we show that $t_1 \geq t_2$. Suppose t_1 is positive so that it satisfies

$$2v(s^M) = v(\phi^{-1}(t_1)).$$

It follows that

$$2TS(s^M) = v(\phi^{-1}(t_1)) + 2\pi(s^M).$$

If t_2 is also positive, than

$$2TS(s^M) = v(\phi^{-1}(t_2)) + \pi(\phi^{-1}(t_2)).$$

Since $2\pi(s^M) > \pi(\phi^{-1}(t_2))$, it follows that $v(\phi^{-1}(t_1)) < v(\phi^{-1}(t_2))$. Then $t_1 > t_2$ follows from that facts that v is decreasing and ϕ is increasing. Moreover, $2v(s^M) > v(c)$ implies that $2v(s^M) > v(c)$. It follows that $t_1 = 0$ implies $t_2 = 0$.

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