

# Risk Aversion in a Dynamic Asset Allocation Experiment <sup>\*</sup>

**Isabelle Brocas**

*University of Southern California  
and CEPR*

**Juan D. Carrillo**

*University of Southern California  
and CEPR*

**Aleksandar Giga**

*University of Southern California*

**Fernando Zapatero**

*University of Southern California  
Marshall School of Business*

## **Abstract**

We conduct a controlled laboratory experiment where subjects dynamically choose their portfolio allocation between a safe and a risky asset. We first derive analytically the optimal allocation of an expected utility maximizer with HARA utility function. We then fit the experimental choices to this model to assess the risk attitude of our subjects. Despite the substantial heterogeneity across subjects, decreasing absolute risk aversion and increasing relative risk aversion are the most prevalent risk types, and we can classify more than 50% of the subjects in this combined category. We also find evidence of increased risk taking after a gain but the effect is small in magnitude. Overall, our robustness tests show that the behavior of subjects is generally well accounted for by the HARA expected utility model. Finally, the analysis at the session level suggests that the behavior of the representative agent is less heterogeneous and closer to (though statistically different from) constant relative risk aversion.

Keywords: laboratory experiments, portfolio allocation, risk aversion, HARA, CRRA.

JEL Classification: C91, D03, D81, G11.

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<sup>\*</sup>We thank the LUSK Center for Real Estate for financial support. Address for correspondence: Juan D. Carrillo, Department of Economics, University of Southern California, 3620 S. Vermont Ave., Los Angeles, CA 90089, USA <juandc@usc.edu>.

# 1 Introduction

Understanding risk is of paramount importance in economics and finance. From a microeconomic perspective, the behavior of entrepreneurs crucially depends on their risk preferences. From a macroeconomic perspective, the risk attitude of the representative agent shapes investment and capital accumulation.

The goal of the study is to design a task that provides a reliable assessment of the risk attitude of individuals in a *controlled yet comprehensive* and *realistic* financial environment. To this purpose, we conduct a laboratory experiment where subjects choose how to invest their wealth between two assets, one safe and one risky, during 15 investment paths. The subject (she) starts a path with an initial endowment, which she allocates between the assets. After observing the returns of the assets, she is asked to reallocate her wealth, and the new returns are observed. This dynamic process lasts for 10 periods, after which her final payoff is recorded and a new path is started with the same initial endowment as the previous path. Overall, each subject makes 150 investment decisions with different levels of wealth.

In this experimental setting, we address the following questions. First, are our subjects similar in their risk attitudes or do we observe substantial heterogeneity? Second, can we fit the data well using a structural estimation of an expected utility model? If so, how general should the specification of the utility function be and what are the most prevalent risk attitudes? Third and related, do we observe frequent and/or severe deviations from neoclassical theory? Fourth, if we analyze the data at the session level, how does the behavior of the “representative agent” compares to that of subjects taken individually?

To answer these questions, we first derive analytically the optimal portfolio allocation of an expected utility maximizer who has a general, two-parameter hyperbolic absolute risk aversion (HARA) utility function. Given this analytical characterization, we can structurally estimate the absolute and relative risk aversion parameters of our subjects using the 150 choices made in the experiment.

We find that our experimental subjects are highly heterogenous. At the same time, some risk attitudes are more prevalent than others. Most individuals increase the total amount of wealth invested in the risky asset as their wealth increases (decreasing absolute risk aversion or DARA). They also decrease the fraction of wealth invested in the risky asset as their wealth increases (increasing relative risk aversion or IRRA). Overall, more than half of our subjects can be confidently classified in the combined DARA-IRRA category, the risk attitude conjectured by Arrow (1971) to be most natural among investors.

Our robustness checks show that the HARA utility function is consistent and has good

predictive power in out-of-sample analysis. We find a statistically reliable relationship between the investment decision and the set of independent variables. Also, when we estimate the parameters using a subsample (either 8 paths randomly chosen or the first 5 periods of all paths) we can predict well the behavior in the complementary subsample. Finally, if we restrict attention to a constant relative risk aversion utility function (CRRA) the accuracy of our estimates suffer significantly for one-third of our population and not significantly for the rest. This provides mixed support for CRRA, the utility function most commonly employed in theoretical and empirical studies. On the one hand, it has a lot of appeal due to its simplicity and convenience. On the other hand, it may yield biased estimates of individual risk attitudes for a non-negligible subset of the population.

We also find some evidence of biases. A few subjects (19%) change their risk taking behavior over time. More significantly, 44% of subjects exhibit a gain/loss asymmetry. Of these, the vast majority (39%) take more risk after a gain and only 5% take more risk after a loss. Overall, many subjects exhibit some anomaly. However, these are small in magnitude which is why, despite their presence, the expected utility model performs well.

Finally, we conduct an aggregate analysis. For each session, we compute the per capita wealth in each period and endow a fictitious “representative agent” with this amount. We find that, just like the majority of our individuals, representative agents are typically best captured by a DARA-IRRA risk type. However, their types are less heterogeneous and closer to constant risk aversion than those of individuals. This suggests that restricting to CRRA functions when we study aggregate behavior may be a reasonable approximation.

Before proceeding to the analysis, we present a brief literature review. Methods to elicit risk attitudes abound in economics. Perhaps the most widely employed technique is the “list method” proposed by Holt and Laury (2002), hereafter [HL]. In this elegant procedure, subjects are offered choices between two lotteries, where each lottery is identified by two possible outcomes and one probability. By varying the stakes and probabilities, one can assess the subjects’ risk attitudes as a function of wealth. The method is fast, intuitive and easy to implement. It offers an excellent, simple measure to compare risk attitudes across individuals. It has been extended in several directions either to improve the precision of estimates (Andersen et al. (2006), Maier and Rieger (2010)) or to obtain a more efficient algorithm (Wang et al. (2014)). Other risk elicitation designs have been proposed by Becker et al. (1964), Binswanger (1980), Hey and Orme (1994), Eckel and Grossman (2008) and Sokol-Hessner et al. (2009) among others.<sup>1</sup>

However, simplicity comes at the expense of a design that is not intended (and therefore

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<sup>1</sup>For surveys of empirical and experimental elicitation procedures and results, see Harrison and Rutström (2008), Charness et al. (2013) and Friedman et al. (2014).

not suitable) to provide a *precise* measure of risk preference of individuals endowed with a general utility function. For example, the [HL] procedure assumes CRRA utility so, by construction, it cannot assess the changes in the percentage of risk taking as a function of wealth. It also provides only interval estimates of the parameter, so it is difficult to assess the fit of the data to the utility specification and to challenge the model. In this regard and as developed above, our methodology has a number of advantages. First and foremost, we can structurally estimate an asset allocation model based on a rich utility function. We can also determine the loss in predictive and explanatory power when we restrict to simpler utility functions. Second, we can measure standard errors of individual estimates and assess the fit of the data. We can also study the structure of the noise and its relation to wealth levels. And third, our dynamic framework is useful for measuring any behavioral anomaly due to repeated exposure to risk. We can detect any gain/loss asymmetry in behavior and determine whether a subject changes her risk attitude over the course of the experiment.

Given the dynamic asset accumulation nature of the experiment, the paper also relates to the experimental literature on portfolio allocation. Levy (1994) proposes a non-structural analysis to study risk attitudes in a market experiment, and finds support for DARA but not for IRRA. In Rapoport (1984) and Rapoport et al. (1988), subjects invest in securities and a safe asset in a dynamic setting, and find evidence in favor of IRRA and against CARA or CRRA. Other related individual asset allocation experiments test whether subjects allocate portfolios efficiently (Kroll et al. (1988), Kroll and Levy (1992), Sundali and Guerrero (2009)). Lastly, Post et al. (2008) study dynamic choices in the gameshow “Deal or No Deal”. They find that the Expected Utility Theory cannot explain the contestants’ decisions well and point that previous outcomes play a significant role in the choices of participants. Contrary to this literature, our main goal is to isolate risk attitudes, which is why we opt for an individual decision-making rather than a market set up. We also provide full information about the design to prevent subjects from forming beliefs we could not observe.

Last, our results on path-dependence of choices and gain/loss asymmetry are related to the literature that highlights behavioral anomalies in choice under uncertainty. Discrepancies between observed behavior and theoretical predictions may come from errors in choices (Jacobson and Petrie (2009)), frequency of feedback (Gneezy and Potters (1997); Thaler et al. (1997)), reference dependent preferences (Koszegi and Rabin (2006, 2007); Abeler et al. (2011); Knetsch and Wong (2011); Ericson and Fuster (2011); Sokol-Hessner et al. (2009)), or disappointment aversion (Choi et al. (2007); Gill and Prowse (2012)) among other reasons. Our design is not intended to test for specific behavioral anomalies,

nor to fit behavioral models. However, like in Thaler and Johnson (1990), we find that prior gains (losses) decrease (increase) risk aversion for many of our subjects.

The article is organized as follows. In section 2, we present the theoretical framework. In section 3, we describe the experimental setting. In section 4, we present the econometric model and the results of the classification analysis. In section 5, we investigate behavioral anomalies. In section 6, we study the explanatory and predictive power of our expected utility model. In section 7, we provide an aggregate analysis of the data. In section 8, we offer some concluding remarks.

## 2 Theory

Consider the following complete markets, continuous-time, dynamic portfolio choice problem. At each instant  $t$ , an agent (she) allocates her wealth  $X(t)$  between two assets, a risky asset  $A$  and a safe asset  $B$ . At  $t = 0$ , her initial wealth is  $X(0) = x_0 > 0$ . The temporal horizon is finite and equal to  $T$ . The agent can reallocate her portfolio at each instant  $t$  until date  $T$ , time at which she enjoys the accumulated wealth  $X(T)$ . Therefore, at each  $t$ , she maximizes the expected utility of wealth at time  $T$ . We assume that the agent's preferences are characterized by the general Hyperbolic Absolute Risk Aversion (HARA) utility function with two parameters,  $\gamma$  and  $\eta$ , first applied by Merton (1971) to a dynamic portfolio allocation. Formally:

$$U(X) = \frac{1-\gamma}{\gamma} \left( \frac{X}{1-\gamma} + \eta \right)^\gamma \quad (1)$$

with the following restrictions:

$$\gamma \neq 1, \quad \frac{X}{1-\gamma} + \eta > 0 \quad \text{and} \quad \eta = 1 \text{ if } \gamma = -\infty$$

This family of utility functions is rich in the sense that it encompasses utility functions with absolute and relative risk aversion that are increasing, constant or decreasing depending on the risk parameters  $\gamma$  and  $\eta$ .<sup>2</sup> The agent exhibits decreasing absolute risk aversion (the empirically most plausible case) when  $-\infty < \gamma < 1$  and constant absolute risk aversion when  $\gamma \rightarrow +\infty$  or  $\gamma \rightarrow -\infty$ . She exhibits increasing, constant and decreasing relative risk aversion when  $\eta > 0$ ,  $\eta = 0$  and  $\eta < 0$ , respectively.

The price of the safe asset  $B(t)$  evolves as follows:

$$dB(t) = e^r B(t) dt \quad (2)$$

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<sup>2</sup>A more general specification of the HARA utility function is:  $U(X) = \frac{1-\gamma}{\gamma} \left( \frac{\beta X}{1-\gamma} + \eta \right)^\gamma$ . In our case, the parameter  $\beta$  is not identified and cannot be estimated.

where  $r > 0$ . The price of the risky asset  $A(t)$  follows a geometric brownian motion with drift  $\mu (> r)$  and diffusion  $\sigma (> 0)$ . Formally:

$$dA(t) = \mu A(t)dt + \sigma A(t)dW(t) \quad (3)$$

where  $W(t)$  is a brownian motion. Let  $\pi(t)$  be the amount of wealth allocated to the risky asset  $A$  at date  $t$ . The wealth  $X(t)$  grows as follows:

$$\begin{aligned} dX(t) &= \pi(t)\mu dt + \pi(t)\sigma dW(t) + [X(t) - \pi(t)] r dt \\ &= [X(t)r + \pi(t)(\mu - r)] dt + \pi(t)\sigma dW(t) \end{aligned}$$

At each date  $t$ , the agent solves the following problem  $\mathcal{P}$ :

$$\begin{aligned} \mathcal{P} : \quad & \max_{\pi} E[U(X(T))] \\ \text{s.t.} \quad & dX(t) = [X(t)r + \pi(\mu - r)]dt + \pi(t)\sigma dW(t) \\ & X(0) = x_o \end{aligned}$$

Given the complete market assumption and the specification of utility and asset returns, our problem has a closed-form solution which we summarize in the next result.

**Proposition 1** *If markets are complete and time is continuous, the optimal amount allocated to the risky asset at date  $t$  when the accumulated wealth is  $X(t)$  is:*

$$\hat{\pi}(t) = \frac{\mu - r}{\sigma^2} \left( \frac{X(t)}{1 - \gamma} + \eta e^{-r(T-t)} \right) \quad (4)$$

Proof. It follows a standard martingale argument (proof available upon request).

The amount allocated to the risky asset depends on the current wealth  $X(t)$ , the investment horizon left  $T - t$ , the returns of the assets and the risk aversion parameters. The model predicts that it increases in the current wealth if the agent exhibits decreasing absolute risk aversion ( $\gamma < 1$ ). The allocation depends on current wealth irrespective of how wealth has been accumulated in the past. Finally,  $\hat{\pi}(t)$  increases (respectively, decreases) as time passes when  $\eta > 0$  (respectively,  $\eta < 0$ ). Note that  $\eta = 0$  corresponds to the CRRA specification, where the agent invests a constant proportion of her wealth in the risky asset irrespective of the level of wealth and the horizon left to invest.

The risk attitude of each agent is defined by both her absolute risk aversion (ARA) and her relative risk aversion (RRA): increasing (I), constant (C) or decreasing (D) in wealth. These are determined by the  $(\gamma, \eta)$  parameter combination of the individual, which from now on will be called her “type”. Equation (4) has predictions for each type in terms of

the amount of wealth invested in the risky asset. First, all types with a DARA component increase the risky investment as wealth increases. Of these, an agent with decreasing relative risk aversion (DARA-DRRA type) is willing to short-sell when her wealth is low ( $\hat{\pi}(t) < 0$  when  $X(t)$  is small). By contrast, an agent with increasing relative risk aversion (DARA-IRRA type) is willing to borrow when her wealth is low ( $\hat{\pi}(t) > X(t)$  when  $X(t)$  is small). Second, types with a IARA component decrease the risky investment as wealth increases. Of these, an agent with increasing relative risk aversion (IARA-IRRA type) will invest a positive amount of wealth in the risky asset only when her wealth is low.

Note that the closed-form solution for the optimal portfolio choice requires a complete markets assumption, otherwise it cannot be characterized analytically. Therefore, if we impose the extra restriction  $\hat{\pi}(t) \in (0, X(t))$ , then the inability to invest unrestrictedly both now and in the future affect current decisions. Nevertheless, some of the qualitative properties of the solution remain. In particular, agents represented by DARA, CARA and IARA utility functions will respectively choose to invest more, the same and less total amounts in the risky asset as their wealth increases. Similarly, agents represented by DRRA, CRRA and IRRA utility functions will respectively choose to invest a larger, an equal and a smaller fraction of their wealth in the risky asset as their wealth increases.

Table 1 summarizes the risk types as a function of  $\eta$  and  $\gamma$  and given the parametric restrictions in the utility function. It shows which types are likely to be constrained and for which wealth levels (indicated by \* and \*\*). It also shows how investment varies with wealth.

	$\gamma < 1$	$\gamma > 1$	$\gamma = -\infty$	$\gamma = +\infty$
$\eta < 0$	DARA-DRRA*	—	—	—
$\eta = 0$	DARA-CRRA	—	—	—
$\eta > 0$	DARA-IRRA**	IARA-IRRA**	CARA-IRRA**	CARA-IRRA**
	$\partial\hat{\pi}/\partial X > 0$	$\partial\hat{\pi}/\partial X < 0$	$\partial\hat{\pi}/\partial X = 0$	$\partial\hat{\pi}/\partial X = 0$

\*  $\hat{\pi} = 0$  for small  $X$  and  $\hat{\pi} = X$  for large  $X$ ; \*\*  $\hat{\pi} = X$  for small  $X$  and  $\hat{\pi} = 0$  for large  $X$

**Table 1.** Risk types as a function of risk aversion parameters.

### 3 Experimental Design

The main objective of the paper is to study the dynamic portfolio choice of agents in a controlled laboratory setting. To this purpose, we design a dynamic investment problem that follows as close as possible the setting of the theory section. Subjects in the experiment allocate wealth between a safe and a risky asset during 15 investment *paths* consisting of

10 *periods* each. The experiment consists of 13 sessions run in the Los Angeles Behavioral Economics Laboratory (LABEL) at the University of Southern California.<sup>3</sup> Each session has between 7 and 10 subjects for a total of 120 recruited subjects, of which 3 are omitted from the analysis due to software malfunction. All subjects participate in three treatments. The first treatment corresponds to the paradigm we study in this paper. Results of the other treatments are reported in Brocas et al. (2014).

Each subject (she) starts each path in period 1 with an endowment of \$3, which she allocates between two assets, a risky asset  $A$  and a safe asset  $B$ . After period 1 ends, each subject earns a return on her portfolio and moves to period 2. She then reallocates her portfolio and earns new returns. This process continues for a total of 10 periods. After period 10, the investment path ends and the subject’s final payoff in that path is recorded. Each subject then moves to the next investment path, where her endowment is reset to \$3. Subjects have 10 seconds to make their decision in period 1 of each path and 6 seconds in periods 2 to 10. They all begin and end investment paths at the same time. Finally, all subject go through 15 paths for a total of 150 choices. Subjects know at the beginning of the experiment the number of paths and periods in path they will go through.

The return of the safe asset  $B$  is 3% while the return of asset  $A$  is drawn from a Normal distribution with mean 6% and standard deviation 55%.<sup>4</sup> The parameters do not change throughout the experiment. The draw of the return is presented in the form of a multiplier, that is, the number that multiplies the allocation to that asset. All participants in a session are subject to the same draws, which makes it possible to analyze the aggregate portfolio of each session (see section 7). At the same time, we make clear to each subject that her return is in no way affected by the allocation decision of the other subject.

Figure 1 provides a screenshot that describes what a subject sees in a given period of a path. Current wealth is represented by the vertical bar positioned above the current period number (period 4 in this example). When gray, the bar is not active and the wealth is not allocated to either asset. Subjects need to click on the bar to activate it and move a horizontal slider to divide their current wealth between assets  $A$  and  $B$ . The upper portion of the bar represents the money invested in  $A$  and the lower portion represents the money invested in  $B$ . The figures on the right side of the bar show the allocation, which can be displayed either in percentage or in dollar terms. To facilitate her reasoning, each subject may change the display of the allocation at any time between percentage in each asset (box labeled “ % ”) and total amount in each asset (box labeled “ \$ ”). After

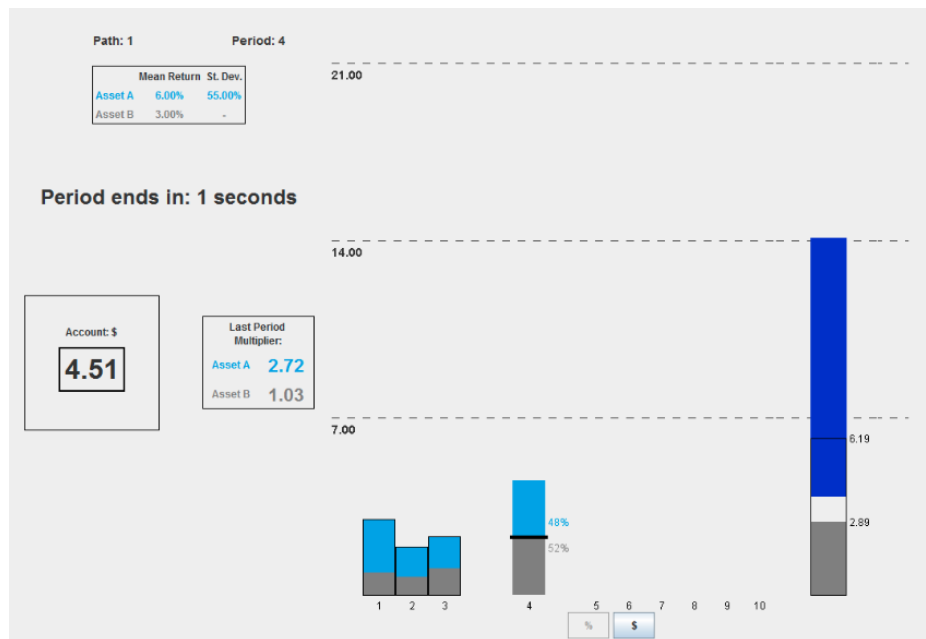
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<sup>3</sup>For information about the laboratory, please visit <http://dornsife.usc.edu/label>.

<sup>4</sup>This (unrealistically high) standard deviation ensures enough volatility in returns for interesting wealth effects and comparative statics.



the period expires, returns are applied and subjects move to the next period. A new bar with a height corresponding to the new wealth appears to the right of the previous one for the new period and becomes inactive again. Subjects need to reactivate it to choose a new allocation, otherwise they earn no interest in that period and their account just carries over. This helps prevent subjects' inertia and a bias towards any status quo allocation. Level of inactivity in our experiment was negligible. Subjects observe bars to the left of the current one (periods 1 to 3 in this screenshot) that reminds them of their past allocations and returns. These bars accumulate up to period 10, and then are reset for the new path. Finally, the left hand side of the screen has a summary information of the main ingredients of the experiment: (i) the current path and period; (ii) a reminder of the mean and standard deviation of returns of assets  $A$  and  $B$ ; (iii) the time left to make a choice in the current period; (iv) the accumulated wealth in the current path; and (v) the multiplier of assets  $A$  and  $B$  in the last period of the current path.



**Figure 1.** Screenshot of path 1 - period 4.

This dynamic wealth allocation problem is challenging and may require substantial learning. To deal with this issue, we employ a highly illustrative 40 minute instructions period using a neutral language with numerical examples, videos, 5 practice paths and a quiz to test the subjects' understanding (instructions can be found in appendix A). In addition, to help with the cognitive strain, we add a projection bar placed on the right

end of the screen (see Figure 1). The projection bar tells the subject what she would expect if she were to keep her current investment strategy until the last period. The bar shows the potential accumulated earnings from asset  $B$  and identifies the 20th, 50th and 80th percentile of the earning distribution from asset  $A$ . As the participant changes her allocation the projection bar automatically adjusts.<sup>5</sup>

At the end of the experiment we collect answers to education, demographic and income related questions as well as their own description of the strategies employed. Each participant receives a \$5 show-up fee and her final earnings in the final period of *one* randomly selected path. Participants are also compensated for participating in the two other treatments. The total length of the experiment is around 2 hours and the average payoff is \$23, with a maximum payoff of \$244.

Note that the experimental design follows closely the theory with two important differences, both introduced for technical reasons. First, choices are made in discrete time, with only 10 decisions per path. Continuous time is difficult to implement in an experimental setting<sup>6</sup> (even if time was continuous, real choices still occur in discrete time given the time it takes to evaluate options and implement decisions). Second, we do not allow our participants to borrow or short sell, which means that markets are incomplete. Borrowing and short selling are difficult to implement experimentally since they may result in taking money away from participants. Our data analysis takes this restriction into account.

## 4 Classification of subjects

Our first objective is to test how well the expected utility theory fits the data and to determine which subjects exhibit systematic departures. We adopt a structural approach and estimate the risk parameters  $(\gamma, \eta)$  of each subject assuming they behave according to the expected utility theory model. We then test for biases. This approach is used to classify our subjects according to their risk type as well as their likelihood to exhibit a behavioral anomaly.

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<sup>5</sup>We carefully explain the function of the bar by simulating potential period-by-period trajectories of wealth coming from a given allocation strategy. The simulation ends each trajectory with a dot, creating a probability distribution of dots. We then draw a parallel between that probability distribution and the projection bar that the subjects see on their screen.

<sup>6</sup>For the challenges and some creative solutions on how to implement continuous time decisions in experimental settings, see Friedman and Oprea (2012).

## 4.1 Econometric model

According to equation (4) –and subject to the above mentioned caveats of incomplete markets and discrete time– expected utility theory predicts that the portfolio allocation and wealth vary over time according to the following system:

$$\begin{cases} \hat{\pi}(t) = \frac{\mu - r}{\sigma^2} \left( \frac{X(t)}{1 - \gamma} + \eta e^{-r(T-t)} \right) \\ dX(t) = [X(t)r + \hat{\pi}(t)(\mu - r)] dt + \hat{\pi}(t)\sigma dW(t) \end{cases}$$

The parameters  $\gamma$  and  $\eta$  can be estimated from the first equation using least squares fitting. Since our data is obtained in discrete time, we consider the discrete version of the model. For each individual, in each path  $i$  and at each period  $t$ , we observe the current wealth  $X_{i,t}$  and the chosen allocation of this wealth to the risky asset  $\pi_{i,t}$ . Let  $F_t = e^{-r(T-t)}$ , our structural econometric model given HARA utility is  $\mathcal{M}^{\text{HARA}}$ :

$$\pi_{i,t} = aX_{i,t} + bF_t + u_{i,t} \tag{5}$$

where  $a = \frac{\mu - r}{\sigma^2(1 - \gamma)}$ ,  $b = \frac{(\mu - r)\eta}{\sigma^2}$  and  $u_{i,t} \sim \mathcal{N}(0, \sigma_u^2)$  is an error term.<sup>7</sup> Given  $a$  and  $b$ , the parameters  $\gamma$  and  $\eta$  are identified. In the next section, we classify the risk attitude of our subjects by fitting this model to their decisions.

Note that a myopic decision-maker would maximize the instantaneous expected utility  $E[U(X(t))]$  at each period  $t$ . This problem has a simple closed-form solution: the optimal allocation in the risky asset is obtained by replacing  $e^{-r(T-t)}$  with 1 in the equilibrium equation of Proposition 1. For our data,  $e^{-r(T-t)} \in [0.7, 1]$ . This value is close enough to 1 to make the myopic model almost indistinguishable from the forward-looking model. Such feature of the design could potentially be a drawback if the objective was to test for forward-looking behavior. However, the problem is sufficiently relevant that a test of optimal choice is interesting independently of whether we consider the myopic or the forward-looking approach. We will therefore not report any results for the myopic model.<sup>8</sup>

Notice that for an accurate estimation, we need enough variation in wealth within subjects. In half of our sample, the 5th and 95th percentile of wealth are around \$1 and \$15, respectively. For the other half of the sample, the range extends from \$1 to \$20. Although these figures are not excessively large, the dispersion is important enough to obtain reliable estimates of absolute and relative risk aversion.

<sup>7</sup>We relax the assumptions on the error term's distribution later (see subsection 4.3.1).

<sup>8</sup>We conducted the same analysis based on the myopic model and we did not find any qualitative changes in the classification of our subjects.

Lastly, our structural model is well specified only if subjects do not systematically invest all their wealth in the safe or the risky asset. In other words and as explained before, the structural model is misspecified for the subjects who are significantly affected by the inability to short sell or borrow. This poses a challenge. On the one hand, treating the data as if all choices are interior amounts to place too much weight on the constrained choices, which biases the interpretation of the parameters and the residuals of the regression. On the other hand, eliminating the constrained choices from the analysis biases the estimated parameters as well. The solution we propose is to classify separately the subjects who hit the bounds often and those who do not.

## 4.2 Constrained choices: willingness to borrow and short sell

Our first task is to determine empirically which subjects are affected by the inability to short-sell (i.e, to set  $\pi_t < 0$ ) and/or borrow (i.e, to set  $\pi_t > X_t$ ). For the large majority of our subjects the pressure to short-sell or borrow is low. At the aggregate level, subjects invest all their wealth in the safe asset 2.2% of the time and in the risky asset 8.3% of the time.<sup>9</sup> At the individual level, there is heterogeneity in behavior. Table 2 shows the distribution of subjects as a function of their likelihood to hit the constraints.

	% trials			Total
	(0%, 10%]	(10%, 20%]	(20%, 100%]	
Hit $\pi_t = 0$ only	10	0	0	10
Hit $\pi_t = X_t$ only	24	3	8	35
Hit $\pi_t \in \{0, X_t\}$	13	11	14	38
$\pi_t \in (0, X_t)$ always	n/a	n/a	n/a	34

**Table 2.** Number of subjects affected by the constraints.

Only 34 subjects never hit a constraint. However, if we combine these subjects with those who hit the constraints no more than 10% of the time we can account for 81 individuals, or 69% of the sample. In what follows, we call these subjects ‘unconstrained’. Of the remaining subjects, 11 would have liked to borrow and 25 would have liked both to borrow and short sell. We call these subjects ‘constrained’.<sup>10</sup>

<sup>9</sup>A choice is defined as non-constrained (interior) when the allocation to the risky asset is bigger than 2% and smaller than 98% of the wealth.

<sup>10</sup>We considered other more conservative thresholds for the division between constrained and unconstrained subjects and found similar results.

## 4.3 Classification of subjects

### 4.3.1 Unconstrained subjects

Our next task consists in estimating the risk aversion parameters  $(\gamma, \eta)$  of the subjects for which the econometric model  $\mathcal{M}^{\text{HARA}}$  is well specified, that is, the 81 unconstrained subjects who either never or rarely hit the short selling and borrowing constraints. Estimating  $\mathcal{M}^{\text{HARA}}$  presents a few challenges. Given our observations are repeated measures for the same subject and given wealth is following a stochastic process, we need to be careful about issues that arise naturally in this time series framework and that may contradict the underlying assumptions required to use the least squares method.

First, the error term should have a constant variance. We run a standard OLS on each individual’s dataset and apply the White test to detect the presence of heteroscedasticity. We find that the variance of the residuals increases with the level of wealth for 73 out of the 81 unconstrained subjects (at the 5% significance level) and is constant for the rest.<sup>11</sup>

Second, error terms should be uncorrelated across periods. We test for serial correlation for each participant by looking at the residuals of the OLS regression, denoted by  $\hat{u}_{i,t}$ . Note first that an error at period  $t - 1$  applied to the amount invested in the risky asset at that period affects the wealth level at period  $t$ . Therefore, regressors are not independent of the error term. To account for this, we use the Breusch-Godfrey test which allows explanatory variables not to be strictly exogenous. Formally, we consider the regression:

$$\hat{u}_{i,t} = \beta_0 + \beta_1 X_{i,t} + \beta_2 F_t + \rho \hat{u}_{i,t-1} + v_{i,t}$$

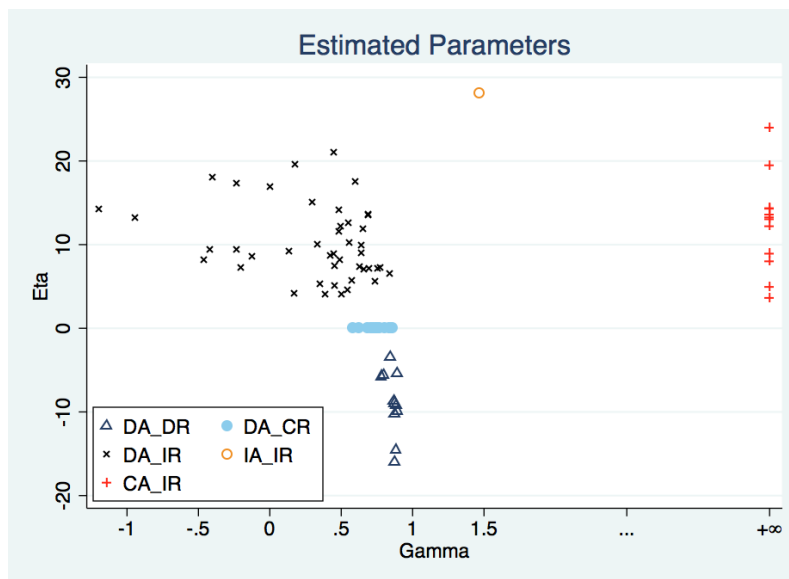
where the  $X_{i,t}$  and  $F_t$  components account for weak exogeneity and  $v_{i,t}$  are assumed to be i.i.d. with normal distribution  $\mathcal{N}(0, \sigma_v^2)$ . We use robust standard errors in our test. We find first order serial correlation ( $\rho > 0$ ) for 63 out of 81 subjects. To correct for heteroscedasticity and autocorrelation, we run the OLS regression with Newey-West standard errors.

Figure 2 displays the estimated  $(\gamma, \eta)$  risk parameters of the 81 unconstrained subjects using the structural model  $\mathcal{M}^{\text{HARA}}$  presented in equation (5). Table 3 reports the relative and absolute risk aversion attitudes based on the estimated parameters. We observe substantial heterogeneity in risk attitudes. At the same time, there is a concentration

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<sup>11</sup>Of the 73 heteroscedastic subjects, 41 choose the percentage display more than 80% of the time and only 24 choose the absolute amount display more than 80% of the time. By contrast, of the 8 homoscedastic subjects, 3 choose the percentage display more than 80% of the time and 4 choose the absolute amount display more than 80% of the time. Since the vast majority are heteroscedastic, the evidence is not sufficient to conclude a positive relationship between reasoning in percentage terms and exhibiting increased volatility with wealth. However, it is an interesting and intuitive possibility worth of future exploration.

in the upper left quadrant: the vast majority of subjects are DARA ( $\gamma < 1$  for 84% of subjects) and IRRA ( $\eta > 0$  for 70% of subjects). Overall, 54% of subjects are willing to increase their total investment in the risky asset and decrease the fraction of investment in the risky asset as their wealth increases. These are the DARA-IRRA subjects ( $\gamma < 1$  and  $\eta > 0$ ) conjectured by Arrow (1971) to be the empirically most plausible types. By contrast, the simple one parameter specifications commonly used in the literature do not capture well the risk attitude of many of our subjects: only 15% of our subjects are CARA ( $\gamma \rightarrow +\infty$ ) and 16% are CRRA ( $\eta = 0$ ).<sup>12</sup>



**Figure 2.** Estimated parameters of the 81 unconstrained subjects

A natural question is to determine how these results compare to the existing estimates in the literature such as, for example, Holt and Laury (2002), hereafter [HL]. To address this issue, we estimate our structural model assuming the familiar functional form:

$$\tilde{U}(X) = \frac{X^{1-\xi}}{1-\xi}$$

used in [HL]. In this case, the solution to the problem  $\mathcal{P}$  described in section 2 is well-established in the literature. Indeed, the agent invests a constant fraction of wealth in the

<sup>12</sup>For our classification, we use CARA and CRRA as the null hypotheses which may over-classify subjects in those categories.

Risk attitude	No. of subjects
DARA-DRRA	11
DARA-CRRA	13
DARA-IRRA	44
IARA-IRRA	1
CARA-IRRA	12
Total	81

**Table 3.** Risk attitude of the unconstrained subjects.

risky asset:

$$\hat{\pi}(t) = \frac{1}{\xi} \frac{\mu - r}{\sigma^2} X(t).$$

Analogously to our strategy in section 4.1, we estimate  $\xi$  from the following econometric model:

$$\pi_{i,t} = cX_{i,t} + \nu_{i,t} \tag{6}$$

where  $c = \frac{\mu - r}{\xi \sigma^2}$  and  $\nu_{i,t} \sim \mathcal{N}(0, \sigma_\nu^2)$  is an error term. Let us call this model  $\mathcal{M}^{\text{CRRA}}$ . We compare our estimates of  $\xi$  to those in [HL]. Because of the way the experiment is designed, [HL] only gives range estimates for the parameter  $\xi$ . Table 4 summarizes the proportion of subjects who fall in each of the six ranges of  $\xi$  in our model ( $\mathcal{M}^{\text{CRRA}}$ ) as well as in the low stakes (\$0.10 to \$3.85, HL-low) and high stakes (\$2 to \$77, HL-high) treatments of Holt and Laury (2002).

Risk Aversion	HL-low	HL-high	$\mathcal{M}^{\text{CRRA}}$
$\xi < 0.15$	.34	.19	.11
$0.15 \leq \xi < 0.41$	.26	.19	.61
$0.41 \leq \xi < 0.68$	.23	.23	.27
$0.68 \leq \xi < 0.97$	.13	.22	.01
$0.97 \leq \xi < 1.37$	.03	.11	.00
$1.37 \leq \xi$	.01	.06	.00
no. of subjects	175	150	81

**Table 4.** Risk estimates: a comparative analysis with Holt and Laury (2002)

Our estimates are substantially more concentrated than in [HL]. Only 11% of our subjects exhibit risk-neutrality or risk-loving preferences ( $\xi < 0.15$ ) as opposed to 34% and 19% in [HL-low] and [HL-high] respectively. Unlike [HL], we also find no evidence

of high ( $0.97 \leq \xi < 1.37$ ) or extremely high ( $1.37 \leq \xi$ ) risk aversion. Overall, we have twice as many subjects as [HL] in the expected range (88% against 49% and 42% in  $0.15 \leq \xi < 0.68$ , which [HL] label as slightly risk averse and risk averse). These differences are important. They are partly due to differences in the design of the two experiments and partly due to the misspecification of the CRRA utility function in our experiment (and possibly in theirs as well). However, the results do not seem in contradiction. They also highlight the advantages of a rich experimental setting to better estimate risk aversion and a two-parameter specification to capture the heterogeneity present in the relative risk aversion of subjects.

### 4.3.2 Constrained subjects

Next, we study the risk attitude of the 36 subjects who, according to the analysis in section 4.2, are constrained by their inability to borrow and short sell. As noted before, the tendency to invest all wealth in the safe or the risky asset should depend on the amount of wealth. We first assess how wealth affects their probability of hitting each bound. More specifically, we estimate a probit regression on the following two models:

$$\begin{aligned}\pi_{i,t}^{max} &= b_0^{max} + b_1^{max} w_{i,t} + \epsilon_{i,t}^{max} \\ \pi_{i,t}^{min} &= b_0^{min} + b_1^{min} w_{i,t} + \epsilon_{i,t}^{min}\end{aligned}$$

where  $\pi_{i,t}^{max}$  takes a value of 1 if  $\pi_{i,t} = w_{i,t}$  and 0 otherwise, and where  $\pi_{i,t}^{min}$  takes a value of 1 if  $\pi_{i,t} = 0$  and 0 otherwise. We establish an effect when  $b_1^{max}$  or  $b_1^{min}$  are different from zero at the 5% significance level.

We find three distinct groups of individuals. The “constrained IRRA” group comprises 28 subjects, who invest their entire wealth in the risky asset when their wealth is low enough ( $b_1^{max} < 0$ ) or invest their entire wealth in the safe asset when their wealth is high enough ( $b_1^{min} > 0$ ) or both. This behavior is consistent with IRRA, although it can also be compatible with risk neutrality for low enough wealth levels.<sup>13</sup> The “constrained irregular” group comprises 7 subjects who exhibit an irregular and volatile behavior with no discernible patterns or statistically significant effects. Finally, the “constrained DARA-DRRA” group comprises 1 subject who invests his entire wealth in the safe asset when his wealth is low enough ( $b_1^{min} < 0$ ) and in the risky asset when his wealth is high enough ( $b_1^{max} > 0$ ), a behavior consistent only with DARA-DRRA. The result (which is the analogue of Table 3 for the constrained subject sample) is summarized in Table 5.

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<sup>13</sup>Of these subjects, 15 are best classified as DARA-IRRA, 10 are best classified as CARA-IRRA, and the remaining 3 are best classified as IARA-IRRA.



Risk attitude	No. of subjects
Constrained IRRA	28
Constrained irregulars	7
Constrained DARA-DRRA	1
Total	36

**Table 5.** Risk attitude of constrained subjects.

Overall, just like for the unconstrained subjects, there is substantial heterogeneity among the constrained subjects. A majority of subjects (78%) exhibit increasing relative risk aversion with more than half of them exhibiting also decreasing absolute risk aversion.

Finally, we use our questionnaire to study the correlation between risk attitude and demographics. We find an over-representation of males in the population of subjects who are affected by the inability to borrow (i.e., hit often the  $\pi_t = X_t$  constraint). More precisely, out of our full sample comprising 49 males and 68 females, 33% of the males (16) and 18% of the females (12) are in the Constrained IRRA group. Among the unconstrained subjects, the distributions of types in the male and female populations are not significantly different.

## 5 Behavioral anomalies

So far, we have assumed that the expected utility model is “correct” and classified the risk attitude of our subjects assuming their behavior is consistent with a HARA utility function. Several studies have reported behavioral anomalies in decision-making under risk and uncertainty. One notable anomaly is the tendency of subjects to repeat choices that have generated gains in the past and avoid choices that have generated losses in the past. In a financial setting, it translates into repeating risky investments after a gain and moving wealth into safe assets after a loss, even when draws are known to be i.i.d. (Thaler and Johnson (1990)). A second and related anomaly is a disproportionate preference to avoid losses relative to acquire gains. One way to describe this behavior is through prospect theory, in which subjects evaluate their options via a reference point (Kahneman and Tversky (1979)). In a financial setting, the reference point can be the current wealth or any other heuristic. From a dynamic perspective, the reference point is likely to change over time, suggesting that a certain degree of time dependence may be observed.<sup>14</sup>

<sup>14</sup>The literature usually uses status-quo or lagged status-quo as natural candidates for the reference point. Koszegi and Rabin (2006, 2007) model the reference point as an expectation. As noted earlier, a recent strand of the literature has tested and supported that hypothesis. Unlike these experiments, our

Our goal in this section is to determine if there are systematic biases in choices due to dynamic considerations rather than to test specific models or fit specific parametric functions. In our dynamic expected utility model, negative or positive shocks at  $t - 1$  affect wealth at  $t$  and therefore the investment decision at  $t$ . The risk attitude of each subject determines how she should respond to positive or negative shocks. To test whether subjects react differently after a positive or a negative shock, or whether time dependence is present, we need to control for any effect that emerges naturally from the model. To do so, we study the residuals of our corrected least squares regression in the group of the 81 unconstrained subjects that are fitted with the  $\mathcal{M}^{\text{HARA}}$  model. We explore their behavior as a function of the path, and the returns obtained in the period immediately before. The results are reported in the next two sections.

## 5.1 Path dependence

To test for path-dependence, we run the following regression:

$$\hat{u}_{i,t} = \beta_0 + \alpha I_{i,t}^{\text{path}>8} + \beta_1 X_{i,t} + \beta_2 F_{i,t} + \rho \hat{u}_{i,t-1} + v_{i,t} \quad (7)$$

where  $I_{i,t}^{\text{path}>8}$  is a dummy variable that takes value 1 if the observation is from a late path (9 to 15) and 0 otherwise. The regression shows no evidence of path-dependence for 63 subjects (at the 5% significance level). Among the remaining 18 subjects, 9 exhibit a positive  $\alpha$  parameter, indicating more risk-taking behavior over time than predicted by the model. The other 9 subjects exhibit a negative  $\alpha$  parameter, indicating less risk-taking over time than predicted by the model.<sup>15</sup> A possible explanation is that subjects learn about their preferences over time and adapt their behavior gradually. To investigate this issue further, we run a regression with squared residuals as the dependent variable in order to assess whether the decisions of subjects become more precise over time. We find that among the 18 subjects with path-dependency, 1 subject commits more mistakes over time (decreasing precision) and none commits fewer mistakes over time. Finally, in order to get a better sense of the magnitude of the path-dependency, we look at the  $\alpha$ -coefficient of the 18 subjects with a statistically significant effect. The largest positive and negative coefficients are  $\alpha = 0.56$  and  $\alpha = -0.51$ , meaning that the error in the estimation due to path-dependency is relatively small. To summarize, 22% of the individuals show statistically significant path-dependency, but small in magnitude.

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design is not suitable for controlling or revealing subjects' expectations. Therefore, we cannot test the endogenous reference point theory without confounds.

<sup>15</sup>The results are similar when we run the regression:  $\hat{u}_{i,t} = \beta_0 + \alpha PT_{i,t} + \beta_1 X_{i,t} + \beta_2 F_{i,t} + \rho \hat{u}_{i,t-1} + v_{i,t}$ , where the independent Path variable PT takes values from 1 to 15.

## 5.2 Gain/loss asymmetry

To check whether subjects react differently after a loss or a gain, we run the regression:

$$\hat{u}_{i,t} = \beta_0 + \alpha I_{i,t}^{gain} + \beta_1 X_{i,t} + \beta_2 F_{i,t} + \rho \hat{u}_{i,t-1} + v_{i,t}$$

where  $I_{i,t}^{gain}$  is a dummy variable that takes value 1 if the subject starts the period  $t$  after a gain at  $t - 1$  and 0 if she starts it after a loss at  $t - 1$  (we use the White-Huber standard errors to account for heteroscedasticity). Our data shows no reaction to previous gains or losses beyond the model prediction for 30 subjects (at the 5% significance level). Among the remaining 51 subjects, the vast majority (46 subjects) exhibit higher residuals after a gain. So, consistent with the findings in Thaler and Johnson (1990), these subjects take more risks after a gain than after a loss. The remaining 5 subjects exhibit the opposite pattern. We then study the magnitude of the  $\alpha$ -coefficient for the subjects with a significant overreaction to previous outcomes. All 5 subjects who take more risks after a loss have a small coefficient:  $|\alpha| < 0.48$ . Among the 46 subjects who take more risks after a gain, 40 have a small overreaction (\$1 or less) and 6 have a more substantial one (between \$1 and \$4). In summary, many subjects (57%) exhibit excessive risk-taking after gains but, except for some notable exceptions (7% of subjects), the overreaction is, once again, small in magnitude.

## 5.3 Classification of behavioral types

Finally, when we compare the two sets of anomalies we find that they mostly involve different subjects. Indeed, 6 subjects exhibit different risk taking behavior in the latter paths of the game (path dependence), 39 subjects exhibit an effect of past period outcomes on current choices (gain/loss asymmetry), and only 12 subjects exhibit both anomalies. Subjects with one or both anomalies are present in all the risk-type categories described in Table 3. The remaining 24 subjects can be confidently classified as expected utility maximizers.

In conclusion, anomalies are prevalent. Residual behavior can be attributed to systematic biases that are not captured by the structural model. On the other hand, anomalies are spread among subjects and small in magnitude, so we can fit the data to the expected utility model reasonably well. In the next section, we provide a more in-depth investigation of the fit of our model.

## 6 Testing the model

The objective of this section is to explore the validity of the HARA specification, both overall and in comparison to the one-parameter CRRA specification. We restrict our attention to the 81 subjects whose behavior can be fitted to our structural  $\mathcal{M}^{\text{HARA}}$  model. To assess the general goodness of fit of our structural model, we first conduct F-tests to determine whether the proposed structural relationship between the risky investment at each period and the set of independent variables is statistically reliable. We find that it is for all 81 subjects. Furthermore, according to the Akaike Information Criterion comparison (AIC),  $\mathcal{M}^{\text{HARA}}$  outperforms  $\mathcal{M}^{\text{CRRA}}$  for all but 7 subjects.<sup>16</sup>

### 6.1 Out-of-sample predictions

We probe the matter further and ask whether we can predict the behavior of our subjects based on the observation of their choices in a subset of trials. More specifically, we pick 8 paths at random to estimate the parameters of the  $\mathcal{M}^{\text{HARA}}$  model and then use the estimates to predict choices on the remaining 7 paths. We repeat the exercise 100 times. For each repetition we calculate the Mean Absolute Error:<sup>17</sup>

$$MAE^{\text{HARA}} = \frac{\sum_{i=1}^7 \sum_{t=1}^{10} |\pi_{i,t} - \tilde{\pi}_{i,t}^{\text{HARA}}|}{70}$$

where  $\tilde{\pi}_{i,t}^{\text{HARA}}$  is the prediction of  $\mathcal{M}^{\text{HARA}}$  on decisions in the 7 validation paths.

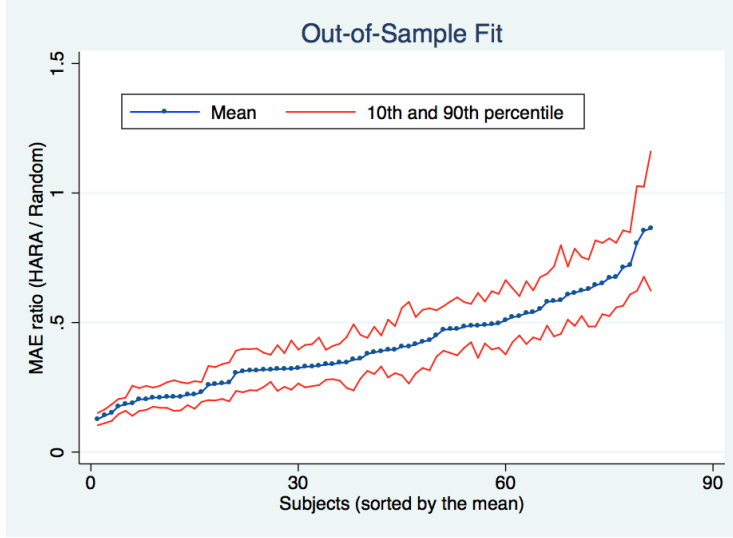
As a benchmark, we first compare the out-of-sample fit of HARA to a model where fit decisions are made at random for the same 7 validation paths. More specifically, we calculate:

$$MAE^{\text{RND}} = \frac{\sum_{i=1}^7 \sum_{t=1}^{10} |\pi_{i,t} - \tilde{\pi}_{i,t}^{\text{RND}}|}{70}$$

where  $\tilde{\pi}_{i,t}^{\text{RND}}$  is an amount drawn from a uniform distribution in  $[0, X_{i,t}]$ . Figure 3 shows the mean, 10th percentile, and 90th percentile of the ratio  $\frac{MAE^{\text{HARA}}}{MAE^{\text{RND}}}$  of 100 repetitions for our 81 subjects, sorted by the mean of the ratios, from smallest to largest. The ratio is intended to describe how much better  $\mathcal{M}^{\text{HARA}}$  explains behavior in comparison to a naïve random model. The ratio is below 1 for all subjects and below 0.5 for 73% of subjects, suggesting (not surprisingly) that for the vast majority the HARA specification performs substantially better out-of-sample than the random specification.

<sup>16</sup>The  $\eta$  parameter of  $\mathcal{M}^{\text{HARA}}$  is estimated to be zero for these 7 subjects, implying de-facto constant relative risk aversion. If we use the Bayesian Information Criterion (BIC), there are 4 more subjects for which  $\mathcal{M}^{\text{CRRA}}$  outperforms  $\mathcal{M}^{\text{HARA}}$ . The estimated  $\eta$  parameter is zero or close to zero for all 11 subjects.

<sup>17</sup>We conducted the same analysis with the root mean square error measure (RMSE) instead of the MAE and obtained similar results.



**Figure 3.** Out-of-Sample Fit (100 repetitions) - HARA vs. Random

Next, we ask a more relevant question: how much predictive power do we lose by considering a simple, one-parameter CRRA specification instead of the richer, two-parameter HARA specification? We follow the same procedure as before with the  $\mathcal{M}^{\text{CRRA}}$  model instead of the  $\mathcal{M}^{\text{HARA}}$  model. More precisely, we calculate the Mean Absolute Error:

$$MAE^{\text{CRRA}} = \frac{\sum_{i=1}^7 \sum_{t=1}^{10} |\pi_{i,t} - \tilde{\pi}_{i,t}^{\text{CRRA}}|}{70}$$

where  $\tilde{\pi}_{i,t}^{\text{CRRA}}$  is the prediction of  $\mathcal{M}^{\text{CRRA}}$  on decisions in the 7 validation paths. Figure 4 shows the mean, 10th percentile, and 90th percentile of the ratio  $\frac{MAE^{\text{HARA}}}{MAE^{\text{CRRA}}}$  of 100 repetitions for our 81 subjects.

The mean of ratios is smaller than 1 for 61 subjects (75%). Half of these subjects have mean ratios below 0.9 and at least 90% of repetitions below 1, which suggests that the improvement of HARA over CRRA is substantial for 37% of subjects and minor for the other 38%. Among the remaining 20 subjects for whom CRRA performs better out-of-sample (25% of the population), the mean ratio is above 1.1 for only 1 subject. Notice also that 11 out of those 20 subjects are specified as having constant relative risk aversion by the HARA model ( $\eta = 0$ ), so the similarity between the out-of-sample predictions of the two models is expected for those individuals. Overall, HARA improves significantly out-of-sample predictions over CRRA for one-third of the sample and performs similarly for the other two-thirds.



**Figure 4.** Out-of-Sample Fit (100 repetitions) - HARA vs. CRRA

## 6.2 Risk type predictions

Our second prediction exercise consists in determining whether the risk type obtained from the data in one sample is consistent with the risk type obtained in the complement. If they are not, it might be because of learning or a preference change. To do so, we divide our sample into “early paths” (first 8 paths) and “late paths” (last 7 paths). We then estimate the risk types of the individuals in each subsample following the same methodology as before, and look at the consistency in the classification of subjects across datasets. The results are reported in Table 6.

Type consistency by paths		Frequency
Consistent	Full - Early - Late	46
Consistent	Full - Early	21
Consistent	Full - Late	11
Inconsistent		3
Total		81

**Table 6.** Consistency in classification - early paths vs. late paths

In line with previous results, we find that subjects are generally consistent between early and late paths. Of the 81 subjects, 46 have the same risk type across all samples, 32 are consistent on the full sample and one subsample, and only 3 subjects have different

risk types in all samples. Table 7 presents the risk aversion attitude of subjects in the full sample as well as the early path and late paths subsamples.

	Full sample	Early paths	Late paths
DARA-DRRA	11	12	10
DARA-CRRA	13	14	23
DARA-IRRA	44	38	33
IARA-IRRA	1	1	1
CARA-IRRA	12	16	14

**Table 7.** Type frequency - early paths vs. late paths

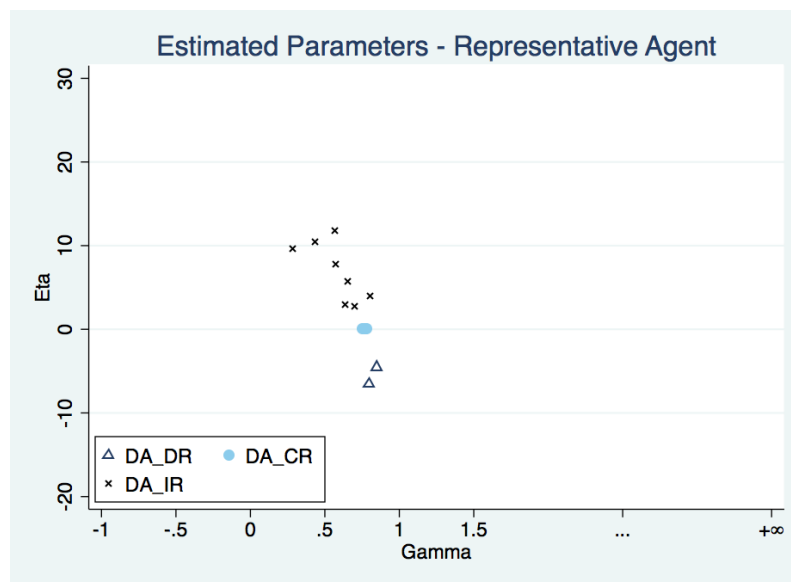
The proportions of the different risk types are, to a large extent, preserved in all samples: there is a majority of DARA-IRRA (between 41% and 54%), virtually no IARA-IRRA, and some representation of the other three types. The most notable difference between samples is the increase in CRRA types at the expense of IRRA types. However, we do not want to excessively emphasize this conclusion as it may be partly due to a statistical effect: with fewer observations per subject in each subsample it may be more difficult to reject the null hypothesis of constant relative risk aversion.

Overall, the expected utility model performs well in our sample. Subjects do not change risk types dramatically over the course of the experiment and it is possible to predict with reasonable accuracy their behavior after observing the choices in the first paths. In appendix B, we perform a similar predictive analysis where the sample is divided between “early periods” (first 5 of each path) and “late periods” (last 5 of each path) and also obtain consistent risk types across subsamples.

## 7 Aggregate risk attitudes

In this section, we perform the same classification exercise as in section 4.3 except that we conduct the analysis at the *session* level rather than at the individual level. Indeed, recall that our experiment consists of 13 sessions with 7 to 10 participants each for a total of 117 subjects. The design permits an objective measure of aggregate wealth because participants are subject to the same shock (the stochastic return of the risky asset in each period of each path is identical for all participants within a session). Instead of summing all the wealth accumulated by subjects in each period, we adopt a per capita specification which allows us to identify the risk preferences of the “representative agent”. This approach makes the results of the individual and session analyses comparable.

By aggregating wealth, the per-capita investment is always interior. We can therefore fit our structural model  $\mathcal{M}^{\text{HARA}}$  to the data. The results of the White test indicates the presence of heteroscedasticity in all sessions. The Breusch-Godfrey test reveals first-order serial correlation in 8 of the 13 sessions. As in the analysis of section 4.3, we correct for both heteroscedasticity and serial correlation using the Newey-West standard errors and estimate the risk parameters  $\gamma$  and  $\eta$  to obtain the risk types of the representative agent in each session. We conduct this analysis using the data from the 81 unconstrained subjects so we can draw a comparison between the individual and the representative agent cases (as a robustness check, we also run the analysis using the full sample of 117 subjects and obtain similar results). Figure 5 displays the analogue of Figure 2 for the session level analysis, that is, the estimated  $(\gamma, \eta)$  risk parameters of the 13 representative agents of our experiment when the 81 unconstrained subjects are considered. For visual comparison we keep the size of the x- and y-axis identical in both figures. Table 8 reports the analogue of Table 3, that is, the relative and absolute risk aversion attitudes based on the estimated parameters, when the analysis is conducted on the unconstrained subjects and on the full sample, respectively.



**Figure 5.** Estimated parameters of representative agents (81 unconstrained subjects)

All sessions exhibit decreasing absolute risk aversion and the considerable majority exhibit also increasing relative risk aversion. There is no evidence of CARA and only some evidence of CRRA (3 sessions). When we add the constrained agents to the analysis,



Risk attitude	No. of sessions	
	Unconstrained (81 subjects)	All (117 subjects)
DARA-DRRA	2	1
DARA-CRRA	3	2
DARA-IRRA	8	10
IARA-IRRA	0	0
CARA-IRRA	0	0
Total	13	13

**Table 8.** Risk attitude of representative agents

relative risk aversion increases but types remain similar. Overall, the comparison between Figures 2 and 5 suggests a similar distribution of types at the individual and session level: in both cases, DARA-IRRA accounts for more than half of the observations and IARA-IRRA are non-existent. We notice, however, two important differences. First, there are no CARA types at the session level (all estimates of  $\gamma$  are below 1). Second, even when we look only at the DARA types, the estimates are substantially more dispersed at the individual than at the session level:  $\eta \in (-17, 21)$  vs.  $\eta \in (-7, 12)$  and  $\gamma \in (-1.2, 0.9)$  vs.  $\gamma \in (0.2, 0.9)$ . The parameter  $\eta$  is, on average, also higher for individuals.

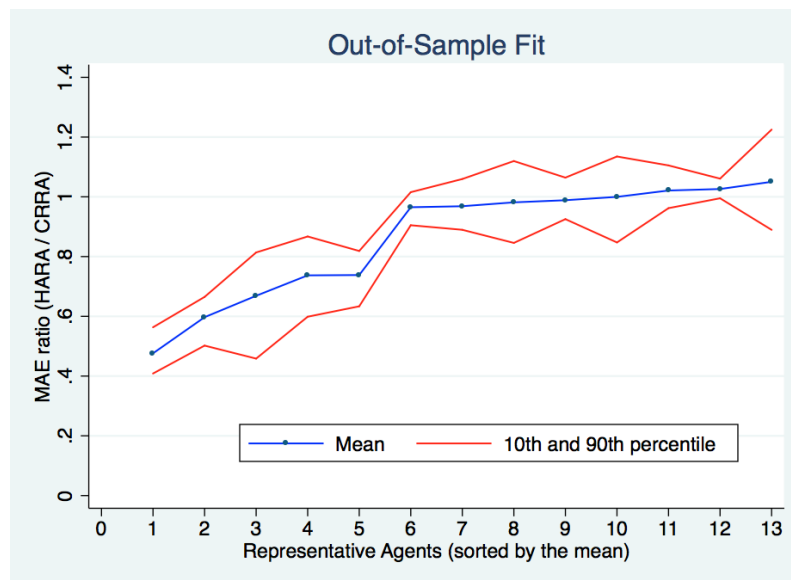
The larger dispersion and higher average of  $\eta$  at the individual level is important, as it suggests that wealth effects are weaker— and therefore that CRRA ( $\eta = 0$ ) is a better approximation— for the representative agent than for the individual data. This results from the combination of two effects. One is purely statistical: by mixing IRRA with DRRA subjects we obtain an aggregate behavior close in magnitude to (even if statistically different from) CRRA. The second is perhaps more subtle: DRRA agents accumulate on average more wealth than IRRA agents and therefore end up having more weight on the average behavior. So, even though there are fewer DRRA than IRRA individuals, their impact in the economy is larger.

We check for evidence of behavioral anomalies at the aggregate level by replicating the analysis of section 5. There is very little evidence of path-dependence: only 3 sessions show an effect and the magnitudes are small. By contrast, we find that all 13 sessions exhibit an asymmetry between gains and losses, with a higher residuals after a gain. The representative agent is therefore taking more risk after a gain than after a loss, consistent with Thaler and Johnson (1990), but the magnitude of the anomaly is, once again, small.

We next explore the out-of-sample predictive properties of the model by performing the same analysis as in section 6.1. Again as a benchmark, we compare HARA to random

choice. For all 13 representative agents, the ratio  $\frac{MAE^{HARA}}{MAE^{RND}}$  is below 0.5 in at least 90% of the repetitions. This means that the improvement of HARA over random choice is greater for the representative agent than for the individual analysis, which implicitly suggests that some of the subjects' deviations cancel each other out.

We then analyze how HARA compares to CRRA. Figure 6, the analogue to Figure 4, shows the mean, 10th percentile, and 90th percentile of the ratio  $\frac{MAE^{HARA}}{MAE^{CRRA}}$  of 100 repetitions for the 13 representative agents. The mean ratio is virtually 1 for 8 sessions and between 0.5 and 0.8 for the other 5 sessions. This means that, just like for the individual analysis, the out-of-sample predictions of the HARA model significantly improve those of CRRA for one-third of the sample and they are very similar for the rest.<sup>18</sup>



**Figure 6.** Out-of-Sample Fit (100 repetitions) - HARA vs. CRRA

When we divide the sample between early paths and late paths, as we did in section 6.2 for the individuals, we find that the representative agent has the same risk type across all samples in 7 sessions (4 DARA-IRRA and 3 DARA-CRRA). Of the remaining 6 sessions, 2 sessions show the same type in the full and early paths subsample and 4 sessions show the same type in the full and late paths subsample. Overall, representative agents are generally consistent across paths, a result which is not surprising given the type-consistency of the majority of individuals in our sample.

<sup>18</sup>When we include the constrained subjects, we find a larger improvement of HARA over CRRA. This is due mostly to the fact that the vast majority of the constrained subjects exhibit IRRA behavior.

To sum up, according to the individual level analysis, estimating a two-parameter HARA utility function rather than a one-parameter CRRA utility function helps improving the estimation of one-third of individuals. The same is true for the session level analysis. The point estimates, however, are substantially more concentrated and closer to constant relative risk aversion in the latter than in the former.

## 8 Conclusion

In this paper, we report the results of an experiment where 117 subjects dynamically choose their wealth allocation. Assuming a HARA utility function, we first construct a structural dynamic choice model which we then use to estimate the absolute and relative risk aversion of the participants. Although technically more complex, this method has the advantage of providing more accurate estimates than traditional risk elicitation techniques.

Even though we find substantial heterogeneity in behavior, decreasing absolute risk aversion and increasing relative risk aversion are the most prevalent subtypes, and we can confidently classify more than half of the subjects in the combined DARA-IRRA category. We also find evidence of increased risk taking after a gain but the effect is small in magnitude, and the behavior of subjects is generally well accounted for by the expected utility model. Finally, our design allows us to perform an aggregate analysis. We find that the representative agent in most sessions is also best represented by DARA-IRRA. However, the estimated risk types at the session level are substantially more concentrated and closer to constant risk aversion than the risk types at the individual level, suggesting that CRRA may be a reasonable approximation for representative agents.

Recent papers have argued that risk attitudes are volatile and difficult to pinpoint (see Friedman et al. (2014) for a survey). Our analysis suggests that if the experimental setting is rich enough, it is possible to accurately estimate (stable) risk preferences. This result is encouraging given the paramount importance for microeconomic theory in understanding risk choices in financial, insurance and environmental settings, just to name a few. At the same time, we also find that in the session level analysis the risk-type estimates are relatively close to (though statistically different from) constant relative risk aversion. Again, this is an encouraging result as it suggests that macroeconomic theories based on the CRRA utility function capture reasonably well the risk characteristics of the representative agent.

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# Appendix

## Appendix A. Instructions

**Note:** The following instructions are accompanied by a slideshow presentation. Slides available upon request.

We are about to begin. Please put your cell phones and other electronic devices in your bag and do not use them until the end of the experiment.

Dear Participants,

Welcome and thank you for coming to this experiment. You will be paid for your participation, in cash, at the end of the experiment. You will remain anonymous to me and to all the other participants during the entire experiment; the only person who will know your identity is the person in the other room who is responsible for paying you at the end. Everyone will be paid in private and you are under no obligation to tell others how much you earned. The entire experiment will take place through the computer terminals.

Let us begin with a brief instruction where you will be given the complete description of the experiment and shown how to use the software. Please, pay attention to the instructions, as it is important for you to understand the details of the experiment. There will be a quiz at the end of the instructions that everyone needs to answer correctly before we can proceed to the actual experiment. Participants who are unable to answer the quiz will not be allowed to participate in the experiment. If you have any questions during the instruction period, raise your hand and your question will be answered so everyone can hear. If any difficulties arise after the experiment has begun, raise your hand, and an experimenter will come and assist you. If you cannot see the entire projection screen, please come forward as it is important for you to see the entire screen.

Today, we will ask you to make investment decisions. Your final payment consists of a \$5 show-up fee plus your investment earnings. Those earnings depend both on the choices you make and on luck. The choices of other participants do not affect your payoff in ANY way, at ANY point in the experiment and your choices do not affect their payoff. The entire experiment is split in 3 parts. I will now give you instructions for Part 1. You will get additional instructions before Part 2 and Part 3.

### PART 1

Let me first summarize the investment process and then we will go through each step in more detail. You will start with an initial amount of money that you will be able to invest in two assets, A and B. You will have 10 periods to invest. At each period you will decide how to allocate your money between the two assets. At the end of each period, you will earn returns from that period's investment in each asset. The two assets will pay differently, and later in the instructions I will explain what to expect from each asset. After period 10, the process ends and the computer will record your final money amount. This process of 10 investment periods is called an investment path. At the start of each path, your money will be reset to the initial amount.

In Part 1 you will complete 15 of these paths. Consequently, there will be 15 final amounts of money, one for each path. The computer will randomly select one of these 15 final amounts. The selected amount will be your payoff for Part 1. Are there any questions? Let me now walk you through the procedure step by step.

**The Initial Endowment:** This is a screenshot of what you will see on your computer at the beginning of each path, that is, in period 1 of each path. In each path you start with \$3, this is

your initial endowment. This amount is displayed on the left side of the screen in the box labeled “Account”. It is also represented by the height of the bar in the middle of the screen. There is a grid in the background to help you get a sense of the bar’s height.

Periods and Timing: As mentioned earlier a path is made of 10 periods, starting at 1 and ending at 10. The sequence is displayed on the bottom of the screen and your current period is displayed in the upper left corner. Each period is an opportunity for you to invest. A period ends when the time runs out. You can see the timer on the left hand side. For the first period in each path you will have 10 seconds to make your investment decision. For the other periods in the path, that is, periods 2 to 10, you will have 6 seconds to make your investment decision.

Investing: Let me show you with a short video how to make your investment.

Step 1. Choose display. To start your investment, you first need to click on one of the two boxes at the bottom, the ones labeled with percentage and dollar signs. These boxes control how your allocation between assets A and B is displayed: in percentages or dollar terms. You have to click one of the boxes in period 1 of each path. You can also change the display anytime simply by clicking on the other box. Select whichever box you find convenient and change it anytime you want.

Step 2. Activate the bar. Now you can activate the investment bar. Click anywhere on the light gray bar to activate it. Notice when the bar is light gray it means that your money is not invested in either asset. If the bar stays that way after the period ends, you will earn zero interest on your money: the same amount will just be transferred to the next period.

Step 3. Choose the allocation. Once you activate the bar you will notice that it is split between two colors: the top is light blue, and the bottom is gray. The top represents the amount of money allocated to Asset A, and the bottom represents the amount of money allocated to Asset B. Now you can see the display I previously mentioned. It shows how much money you have allocated to A and B either in dollar or in percentage terms. This example shows the dollar display. You can change the allocation between A and B in two ways: by holding the horizontal bar and moving it up or down or by clicking on the bar, as you can see in the video. Once you are satisfied with the allocation wait until the period ends.

Step 4. Proceed to the next period. When the period ends, a new gray bar will appear showing you the new amount. Here is the transition from period 1 to period 2. Your new amount will be the sum of the money you earned on both assets A and B and it will be shown on the left where your initial money amount was displayed. The new height of the bar will also represent this amount. Be aware that the background grid can be re-scaled to accommodate changes in the bar, so pay attention to the figures written on the grid. The last period’s bar will become inactive but you will still be able to see your past allocation between assets A and B. Remember that you need to activate the bar and choose an allocation between A and B at every period, otherwise you earn no interest. Here is a period 2 allocation process and the transition to period 3. Notice how I changed the display from dollars to percentages. This process continues until period 10. After period 10, the path ends. Here is a screenshot of one path end. Your final amount will be shown in the box on the left and by the height of a green bar on the right. A message will appear informing you that the path ended. You need to click the “OK” button to continue. A new path will start shortly thereafter.

Assets A and B: Let me show you what to expect from the investment in each asset. In the upper left corner of your screen there is a box that reads “Asset A: mean return 6%, standard deviation 55%”; “Asset B mean return 3%”. These numbers show how your investment in each asset grows and they will not change during the entire experiment.



Asset B: The 3% next to Asset B in the box means that, once the period ends, the amount allocated to Asset B will grow by 3% for sure. The interest rate of 3% will not change throughout the duration of the experiment. A reminder: money in Asset B is represented by the bottom, GRAY portion of your active bar. Here is an example: if you have 2 dollars invested in B you will have 2 dollars and 6 cents in the next period. If you keep that money in B you will then have 2 dollars and 12.2 cents the period after. You can think of your money in Asset B being multiplied by 1.03. Note that 2 dollars is just an example. In the experiment you can choose any allocation you want provided it does not exceed your total amount.

Asset A: Contrary to asset B, your return on asset A is uncertain. Technically, the return on asset A has a Normal distribution with mean 6% and standard deviation 55%, as shown in the upper left box. This means that asset A grows by 6% on average. However, it may be more or it may be less. In particular, the growth rate could be negative. In this case the money you invested in Asset A will shrink. Although the return can be negative, the amount of money you hold on asset A can never go below zero. A reminder: money in Asset A is represented by the top, LIGHT BLUE portion of your active bar.

Another way to think about the return on this asset is that the amount you put in asset A will be multiplied by some positive number. On average, this number will be 1.06 which corresponds to a 6% growth. Let us call this number a multiplier. If the multiplier is less than 1, it means that your investment in Asset A shrinks. For example, if you allocate \$2 to asset A and the multiplier turns out to be 0.8, you will have \$1.6 in the next period. If the multiplier turns out to be 1.5, you will have \$3 the next period. Here is a chart showing the probability of your multiplier being in a given range. With 20% chance it will be somewhere between 0 and 0.67. With 30% chance it will be somewhere between 0.67 and 1.06. With another 30% chance it will be somewhere between 1.06 and 1.7. Finally, with 20% chance it will be above 1.7. Once the period ends and you receive the returns on your assets, the box on the left marked “Last Period Multiplier” will show what turned out to be the multiplier for asset A in that period. The box will show always 1.03 as the multiplier for asset B.

Projection Bar: The returns from asset A obtained after several periods depend on many factors. In order to help you get an idea of the range of outcomes, we placed a projection bar at the end of the screen. Let me explain how the projection bar works. Suppose for example that in the first period you invest \$2 in Asset A. If you keep the returns on that same asset, how much money will you have at the end of the 10th period? Observe what happens on the left hand side of the graph. It is a simulation of your return. The vertical axis represents dollars and the horizontal axis the periods. It begins with 2 dollars in the first period and it ends after 10 periods. Here is one potential final amount of money. But it can also be this. Or this. Or this. Notice that each time a path ends, we keep track where it lands by adding a dot on the right graph. Each dot represents a possible final amount of money. If we run enough paths, all with \$2 invested in asset A, we will get a bunch of dots on the right end. The more dots each dollar region has, the more likely your amount of money will end up there. And that’s exactly what the bar represents: the likelihood of your earnings ending up in a certain amount.

Now look at the example in the picture. It is period 4. Look at the projection bar. For the current investment strategy, the lower gray part is the projection of how much you will earn on asset B if you don’t change the allocation between assets until the end of the path. In this case, you will earn 1.89 dollars on asset B. This amount is for sure since there is no uncertainty on this asset. The upper part shows the projected earnings on asset A if you don’t change the allocation between assets until the end of the path. They correspond to the dots shown in the video. There

is a 20% chance that the final amount lands in the white area above the gray one, a 60% chance that it lands in the dark blue area and another 20% chance that it lands above the dark blue area. Finally, there is a thick line showing the median, in this example, 16 dollars and 64 cents. This means that with a 50% chance your final amount will be somewhere below that number and with a 50% chance it will be somewhere above that number.

Notice also from our demonstration that probabilities are different within a segment. For example, receiving an amount above the dark blue area has a 20% chance, but within this 20% it is more likely to be close to the dark blue area than further away. In other words, it is more likely to get this payoff [point to the slide] than this payoff [point to the slide], although both are possible. You can see this point more clearly on the frequency table. Based on the number of circles, it is more likely that your payoff will end up here [point to the slide] rather than [point to the slide] here, even though both of these areas correspond to the 20% region above the projection bar.

#### Important Points:

1. The projection bar shows the likelihood of different final earnings at the end of the path ASSUMING the amount you receive from each asset is reinvested in the same asset in all the following periods. However, you can change the allocation between assets at every period.
2. At each period, the projection bar recalculates the probabilities. If you move the cursor up and down within a period, the bar shows instantly the new projection.
3. The multipliers on asset A are independent across periods. In other words, the multipliers of previous periods will in no way impact the multiplier in the current period. For example, if the multiplier in a previous period was very high, it does not mean it will be high again. The new multiplier will simply follow the rules of uncertainty described before.
4. All the participants start and end the paths at the same time. The clock starts as soon as the screen appears, so pay attention.
5. The multiplier for asset A in each period is the same for all participants. So, for example if the computer chooses 2 as a multiplier in period 4, it means that all participants will have their investment in asset A doubled.

Are there any questions? If not, let us proceed to 5 practice paths. What you earn on these paths will not count towards your payment; these are meant only for you to familiarize yourself with the entire process of allocating money between assets A and B. Feel free to explore as many investment strategies as possible to better understand the different options.

Please double click on the icon on your desktop that says ABC STUDY. When the computer prompts you for your name, type a 4 digit number that you can easily remember. Please do not forget the number you typed. Then click SUBMIT and wait for further instructions.

Pay attention to the screen. The first practice path will be starting soon. Focus on understanding how to choose the display between percentage and dollars, how to activate the bar, and how to change the allocation between assets. Reminder: Once a path ends, you need to click the OK button in order to proceed to the next path.

[START game] [Complete practice path 1]

You have now completed practice path 1. Are there any questions? Let's proceed to practice paths 2 and 3. Now try to explore different investment strategies to get a good understanding of the investment process.

[Complete practice paths 2 and 3]

You have now completed 3 practice paths. Are there any questions? If not, we will proceed to a short quiz. Please pay close attention to answering the questions, as you will not be permitted to continue with the experiment if you do not answer the questions correctly. Raise your hand if you have any question during the quiz.

[Complete quiz]

You have now completed the quiz. Let us proceed to the last 2 practice paths.

[Complete practice path 4 and 5]

You have now completed practice paths 4 and 5. Are there any questions? Before we start please write down your ID on your record sheet in front of you. You will locate your ID on the left side of your window bar. You will have to present the record sheet to get paid at the end of the experiment. Did everyone right their IDs down?

Let me remind you how you will be paid for Part 1. At the end of the experiment, the computer will randomly select one of the 15 paths and you'll be paid the final amount you earned in that path. Are there any questions? If there are any problems or questions from this point on, raise your hand and an experimenter will come and assist you. We are ready to start the experiment. Please pull out your dividers.

QUIZ (accompanied by a display print-out):

1. Look at the display on the paper in front of you. What is the current period?
  - (a) 1
  - (b) 2
  - (c) 6 (correct)
  - (d) 8
2. Had the person not chosen any allocation between assets A and B, how much would she have in the next period?
  - (a) \$1.44
  - (b) \$4.16
  - (c) \$5.60 (correct)
  - (d) \$7.29
3. In this period, how much has the person invested in Asset A?
  - (a) \$5.60
  - (b) \$4.16 (correct)
  - (c) \$1.44
  - (d) \$0.81
4. Assume this person keeps reinvesting the returns of asset A in A and the returns of asset B in B until the end of the path. Given the current allocation, how much money will this person have in asset B after the path ends?

- (a) \$1.67 (correct)
  - (b) \$1.44
  - (c) \$1.03
  - (d) \$7.29
5. Assume this person keeps reinvesting the returns of asset A on A and the returns of asset B on B until the end of the path. Given the current allocation, how much money will this person have in asset A after the path ends?
- (a) \$7.29
  - (b) \$5.60
  - (c) \$18.00
  - (d) Cannot be determined with certainty (correct)
6. Forget about the display. Imagine you invest \$1 in Asset A and \$1 in Asset B and suppose the multiplier on Assets A and B are 2.00 and 1.03 respectively. How much money will you have in the next period?
- (a) \$3.03 (correct)
  - (b) \$4.00
  - (c) \$5.03
  - (d) Cannot be determined from the information given.

## Appendix B. Alternative predictive analysis

We perform the same analysis as in section 6.2 except that we divide the sample into early periods (first 5 of each path) and late periods (last 5 of each path). Notice that wealth is likely to be lower in the early periods. We first confirm that intuition: wealth is on average \$6.7 in the last five periods compared to \$3.9 in the first 5 periods for the 81 subjects in  $\mathcal{M}^{\text{HARA}}$ . With this potential source of differences in mind, we present in Tables 9 and 10 the analogue information of Tables 6 and 7 for the new subdivision of samples.<sup>19</sup>

Type consistency by periods		Frequency
Consistent	Full - Early - Late	39
Consistent	Full - Early	9
Consistent	Full - Late	25
Inconsistent		8
Total		81

**Table 9.** Consistency in classification - early periods vs. late periods

	Full sample	Early periods	Late periods
DARA-DRRA	11	11	8
DARA-CRRA	13	27	20
DARA-IRRA	44	31	37
IARA-IRRA	1	1	1
CARA-IRRA	12	10	14
Not classified	0	1	1

**Table 10.** Type frequency - early periods vs. late periods

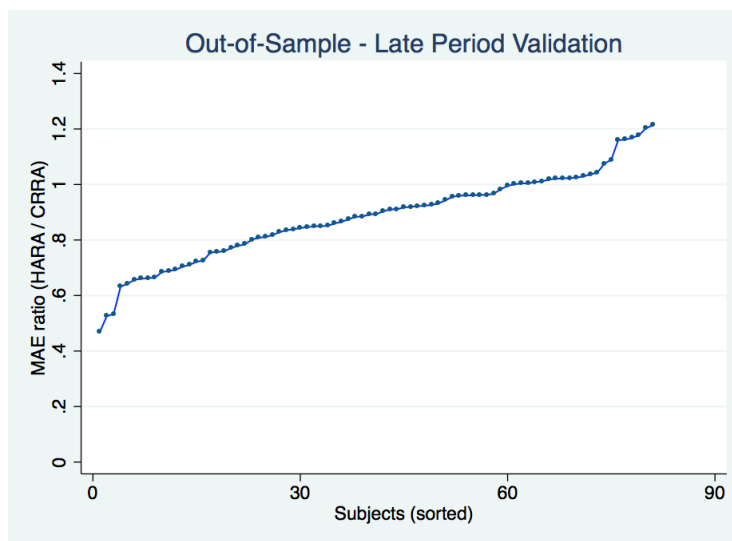
As before most subjects are consistent between the full sample and at least one subsample, though the number of inconsistent subjects is slightly higher than for the early/late paths division (8 vs. 3). Risk attitudes are also similar between samples, with a large representation of DARA-IRRA and an absence of IARA-IRRA. Also as before (and with the same caveat) there is an increase in CRRA subjects at the expense of IRRA subjects.

We also conduct a similar out-of-sample prediction analysis as in section 6.1, except that we estimate parameters on the first 5 periods of all paths and predict choices in the last 5 periods of all paths. This is different than before not only in that the sample

<sup>19</sup>The type “Not classified” in Table 10 corresponds to individuals for which both the  $a$  and  $b$  coefficients in equation (5) are zero.

division is based on periods rather than paths, but also in that we do not take subsamples randomly. The purpose is to test whether extrapolating choices based on risk attitudes elicited in trials with low wealth is meaningful to explain decisions in trials with high wealth.

We get that all but one subject have a ratio  $\frac{MAE^{HARA}}{MAE^{RND}}$  smaller than 1 and, as before, more than two-thirds of subjects have a ratio smaller than 0.5 indicating, again not surprisingly, a large improvement of HARA over random choice. We then present in Figure 7 the analogue of Figure 4 to the new sample division, that is, the the ratio  $\frac{MAE^{HARA}}{MAE^{CRRA}}$  sorted by subjects from smallest to largest.



**Figure 7.** Out-of-Sample Fit - HARA vs. CRRA in early vs. late periods

The results are remarkably similar to those obtained in section 6.1. One-third of subjects exhibit a considerable improvement of HARA over CRRA whereas the other two-thirds are similar. The most notable difference is the existence of a few subjects (6) for which CRRA performs better than HARA. Overall, the results confirm those in section 6: the estimated types are consistent across subsamples (even when we use “low” wealth estimates to predict “high” wealth choices) and a general utility function helps in the estimation for one-third of the individuals.